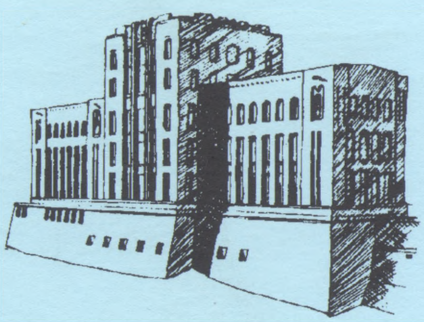
ASSIGNMENT PROBLEMS ON SEDIMENT-TRANSPORT LECTURES AT IOWA INSTITUTE OF HYDRAULIC RESEARCH

Compiled by

The Late John F. Kennedy
and
Tatsuaki Nakato

For the Course
053:173: Mechanics of Sediment Transport

Department of Civil and Environmental Engineering
College of Engineering
The University of Iowa
Iowa City, Iowa 52242



IIHR MONOGRAPH No. 117

Iowa Institute of Hydraulic Research
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Iowa City, Iowa 52242-1585

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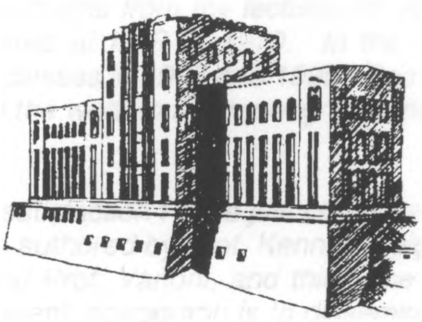

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PREFACE

From 1966, when he came to Iowa from MIT, until 1991, when he died, at age 57 from Multiple Myeloma, Prof. John F. Kennedy taught "Mechanics of Sediment Transport" at the College of Engineering's Iowa Institute of Hydraulic Research (IIHR), The University of Iowa. He demonstrated a profound knowledge and an incredible enthusiasm for the subject. His lectures were so well-known all over the world that many foreign students came to Iowa just to attend his classes.

I took Prof. Kennedy's class for the first time in the spring of 1971 when I arrived from Japan. I vividly remember that long, wintry semester, working through a hefty load of assignments, and being impressed by the fantastic, sometimes humorous nature of the assignment problems. Prof. Kennedy had a desire to compile his assignment problems in a systematic manner, and he finally prepared an unbound problem booklet in 1972. Included therein, in addition to many problems he developed, were three problems developed by the late Prof. Arthur T. Ippen (Massachusetts Institute of Technology, Cambridge, Massachusetts, U.S.A.) and fifteen problems authored by Prof. Vito A. Vanoni (California Institute of Technology, Pasadena, California, U.S.A.). Prof. Kennedy apparently collected those problems through his academic association with them before arriving at IIHR.

Prof. Kennedy revised the booklet in 1975. Revisions 2 and 3 followed in 1989 and 1991. All the problems he compiled were typewritten and the figures were pencil-drawn. Following Prof. Kennedy's death, I started teaching his course in 1993. I began re-typing all the problems that he compiled and re-drawing all the figures using a CAD program on a PC. I added my own assignment problems to his collection in 1994, including some of the old problems from my lectures in "Advanced Topics in Sediment Transport," which were offered at IIHR in 1979. In the meantime, I distributed draft problems to students in my classes in 1993 and 1995. Some IIHR graduates have since heard of my work to expand the work to this monograph and asked for copies of it, and I have been happy to oblige.

With great personal satisfaction I have just completed my work. This monograph contains sixty-four problems authored by Prof. Kennedy, eighteen problems authored by both the late Prof. Ippen and Prof. Vanoni, and thirty-one problems developed by me. The sole intention of the present monograph is to disseminate this fascinating collection by Prof. Kennedy on sediment-transport problems to the hydraulics community worldwide. My dream would come true if any one of Prof. Kennedy's former students or anyone interested in sediment-transport mechanics would utilize this monograph in the classroom to enhance his or her educational endeavor in sediment transport.

In order to acknowledge the authorship of the problems, special notations are used in the problem numbers: problems designated by a ***K*** *were prepared by the late Prof. John F. Kennedy; those including* ***I*** *in the problem number were authored by the late Prof. Arthur T. Ippen; those identified by a* ***V*** *were authored by Prof. Vito A. Vanoni; and, those designated by an* ***N*** *were prepared by Tatsuaki Nakato.*

With fond memories of Prof. John F. Kennedy,
Tatsuaki Nakato
8 February 1998

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I. SEDIMENT PARTICLE CHARACTERISTICS

<PROBLEM K-1P>

A sphere, 18 inches in diameter and weighing 85 pounds, is released in water with kinematic viscosity $\nu = 10^{-5}$ ft^2/sec. Find its terminal fall velocity, w.

Note: Volume of a sphere = $\frac{4}{3}\pi R^3$, where R is the radius of a sphere.

<PROBLEM K-2P>

Find the fall velocity in water at 24 °C of a Bakelite particle with a sieve diameter of 0.8 mm and a shape factor of 0.7.

<PROBLEM K-3P>

You are stranded on the planet Mitiap. While waiting for NASA to retrieve you, you undertake a study of transport of Mitiap sediments. You construct a set of sieves from pieces of your demolished space capsule and rocket, and perform a sieve analysis. However, you have no log-probability paper, so you make your own. Give the coordinate positions for a normal distribution horizontal scale that extends from 0.01 to 99.99 in a distance of 10 in. or 25 cm and a vertical logarithmic scale that includes two cycles in 8 in. or 20 cm. (Note: Tables of the normal distribution, or integrated error function, can be found in most books of mathematical tables, and in *Introduction to Mathematical Statistics*, by Hoel, p.315.)

<PROBLEM K-4P>

The uniformity coefficient, C_u, of a sediment sample is defined as

$$C_u = \frac{D_{60}'}{D_{10}'} \tag{1}$$

where D' is the sieve diameter. Obtain an analytical relation between σ_g and the uniformity coefficient for sediments with log-normally distributed sieve diameters. Then, plot the graphical relation between C_u and σ_g.

<PROBLEM K-5P>

(1) A sieve analysis yields the data given in the table below. Find D_{50} and σ_g for the sieve diameters, and the median and geometric standard deviation for the fall velocity distribution.

(2) Can the distribution be represented as an ordinary normal distribution?

Tyler Mesh (openings per inch)	**Sieve Opening (mm)**	**Amount Retained (gram)**
32	0.495	0.85
35	0.417	1.56
42	0.351	3.88
48	0.295	3.82
60	0.246	5.35
65	0.208	5.69
80	0.175	4.31
100	0.147	5.06
115	0.124	2.37
150	0.104	1.16
170	0.088	0.21
200	0.074	0.12
pan	--------	0.04

<PROBLEM K-6P>

A composite sediment is to be made consisting of two parts of the sediment described in Problem K-5P, and one part of the sediment described below. Find the frequency distribution of the sieve diameters of the composite sediment.

Sieve Opening (mm)	Amount Retained (grams)
0.053	0.66
0.061	2.22
0.074	6.32
0.088	9.98
0.104	22.41
0.124	17.32
0.147	21.92
0.175	9.61
0.208	8.39
0.246	2.71
0.295	0.74
0.351	0.16
0.417	0.00
0.495	0.00

<PROBLEM K-7P>

For the distribution of sieve diameters given in Problem K-5P, find graphically the cumulative distribution of fall velocities in water at 24 °C, using Figs. 2.10 and 2.2 (ASCE Sedimentation Manual No. 54). Compare the values of w_{50} and σ_w obtained graphically with those yielded by the analytic procedure developed by Kennedy and Koh ("The relation between the frequency distributions of sieve diameters and fall velocities of sediment particles," *Journal of Geophysical Research*, 66, 12, December, 1961, pp. 4233-4246).

<PROBLEM K-8P>

You are exploring the newly discovered planet Mitcit. You find that the natural sediments on Mitcit are spherical (Be careful not to spill any on the floor of your space capsule!) and have a frequency distribution given by

$$f(D) = \frac{\pi}{2(D_M - D_m)} \sin \frac{\pi(D - D_m)}{(D_M - D_m)} \qquad (1)$$

where D_M and D_m are respectively the largest and smallest particle sizes present in the sediment. You have also found that in the Mitcit gravity field, the fall velocity Reynolds number of the sediment particles in Earth fluids is less than about 0.5.

(1) Why should you avoid spilling any particles?

(2) Derive the distribution function (i.e., the cumulative frequency function) for fall velocities of Mitcit sediments in Earth fluids. Use Stokes' law to calculate fall velocities.

(3) Derive a relation between the uniformity coefficient (D_{60}/D_{10}), and D_M and D_m for Mitcit sediments.

(4) For the distribution of diameters given above, does

$$\frac{D_{84.1}}{D_{50}} = \frac{D_{50}}{D_{15.9}} \qquad (2)$$

behave as in the case for a log-normal distribution?

(5) What does the spherical shape suggest about the weathering process on Mitcit?

<PROBLEM K-9P>

(1) Show that

$$\frac{\partial w}{\partial \nu} = \left(\frac{m}{m-2}\right)\frac{w}{\nu} \quad \text{and} \quad \frac{\partial w}{\partial (s-1)} = \frac{1}{(2-m)}\frac{w}{(s-1)} \tag{1}$$

where

$$w^2 = \frac{4}{3}(s-1)\,\frac{gD}{C_D} \tag{2}$$

(2) Also, prove that

$$m = -\frac{d\,(\log C_D)}{d\,(\log R)} \tag{3}$$

where

$$R = \frac{wD}{\nu} \tag{4}$$

[Note]:Refer to "The relation between the frequency distributions of sieve diameters and fall velocities of sediment particles," by Kennedy and Koh (*Journal of Geophysical Research*, 66, 12, December, 1961, pp. 4233-4246)

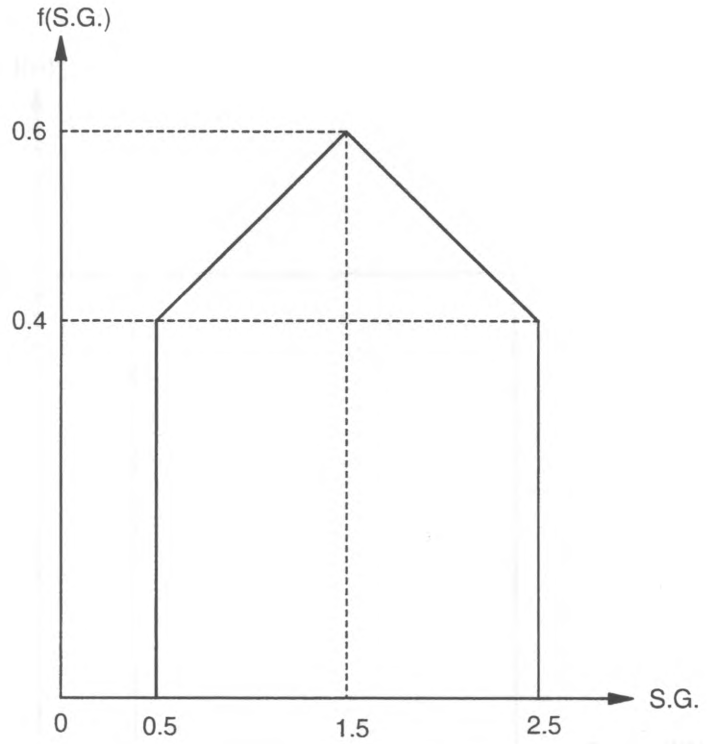
<PROBLEM K-10P>

Find the distribution of fall velocities for a sediment which has log-normally distributed sedimentation diameters, D, with D_{50} = 0.3 mm and geometric standard deviation σ_g = 1.40. Assume that the water temperature is 30 °C (ν = 0.00780 cm^2/sec). Refer to the paper by Kennedy and Koh (1961).

Note: $$\frac{1+m}{2-m} = 1.70R^{-0.120}$$

<PROBLEM K-11P>

Spherical particles with a diameter of 0.305 mm (0.001 ft) fall in water with kinematic viscosity of 10^{-5} ft^2/s. The densities of the particles are distributed as shown. Find the frequency distribution of the fall velocities. Assume that Stokes' law is applicable.



<PROBLEM K-12P>

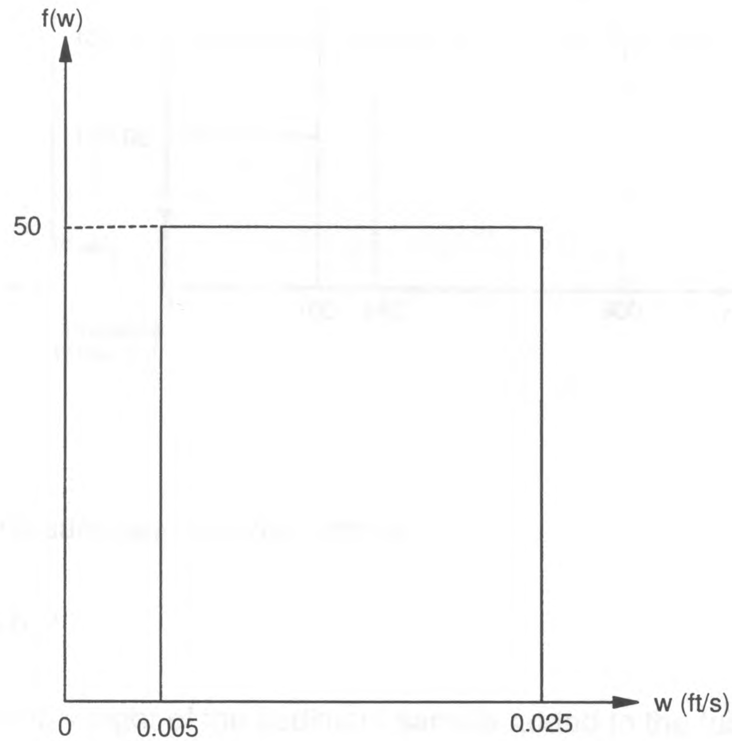
Now consider Problem K-11P with specific gravity, S.G. = 1.5, but D variable.

(1) Would it be possible to find a frequency distribution of diameters, f(D), that would produce the same f(w) as obtained in Problem K-11P? Why?

(2) Find the frequency distribution of diameters that gives the same frequency distribution of fall velocities as the particles with S.G. > 1.5.

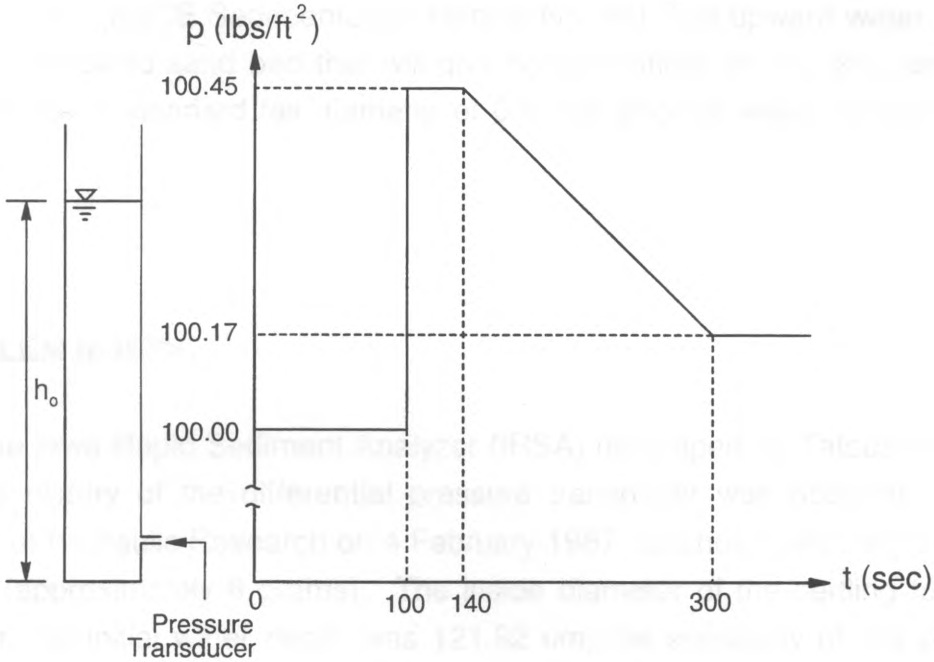
<PROBLEM K-13P>

Find the frequency distribution of the sediment particles, f(D), that produces the frequency distribution of the fall velocities, f(w), shown below. Assume Stokes' law to be applicable, and take S.G. (specific gravity) = 2.5 and ν (kinematic viscosity) = 10^{-5} ft^2/sec.



<PROBLEM K-14P>

A method proposed by Woods Hole Oceanographic Institute for measuring the distribution of particle fall velocities consists of placing a small sample of the sediment in the device shown, called a settling tube, and recording the pressure variation as the sediment settles past the piezometer tap. Take the cross-section area of the tube to be A = 0.10 ft^2. The fluid is water with kinematic viscosity $\nu = 1.00 \times 10^{-5}$ ft^2/s. The sediment sample is added at t = 100 sec.



For the time-pressure record shown above:

(1) What is h_o?

(2) What is the weight of the sediment sample added to the tube?

(3) What is the volume of sediment solids placed in the tube, and what is the specific gravity of the sediment particles?

(4) Find the frequency distribution of fall velocities.

(5) Assuming that the particles are spherical, what are the maximum and minimum diameters present in the sediment?

<PROBLEM K-15P>

Using Fig. 2.5 (ASCE Sedimentation Manual No. 54), find upward water velocity through a fluidized sand bed that will give concentrations of 1%, 2%, and 4% if the sand has a standard fall diameter of 0.1 mm and the water temperature is 24°C.

<PROBLEM N-16P>

Using the Iowa Rapid Sediment Analyzer (IRSA) developed by Tatsuaki Nakato, the time history of the differential pressure transducer was obtained at Iowa Institute of Hydraulic Research on 4 February 1987, as shown, with a glass-bead sample (approximately 6 grams). The inside diameter of the settling tube was 7.00 cm; the initial water depth was 121.92 cm; the sensitivity of the pressure transducer was 15.27 volts/cm H_2O; and the water temperature was 19.5 °C.

(1) What are the true weight and the volume of the sample tested? Can you say that glass beads are quartz?

(2) Obtain the minimum and the maximum fall velocities of the sample particles.

(3) From the given data and curve, obtain the frequency distribution of fall velocities (do not take more than 15 discrete data points from the graph).

(4) Assuming that the particles are spherical, obtain the size distribution curve, and determine the median diameter and the geometric standard deviation of the particles. The manufacturer claims that the particles are between 0.90 mm and 1.23 mm. Do you agree with the claim?

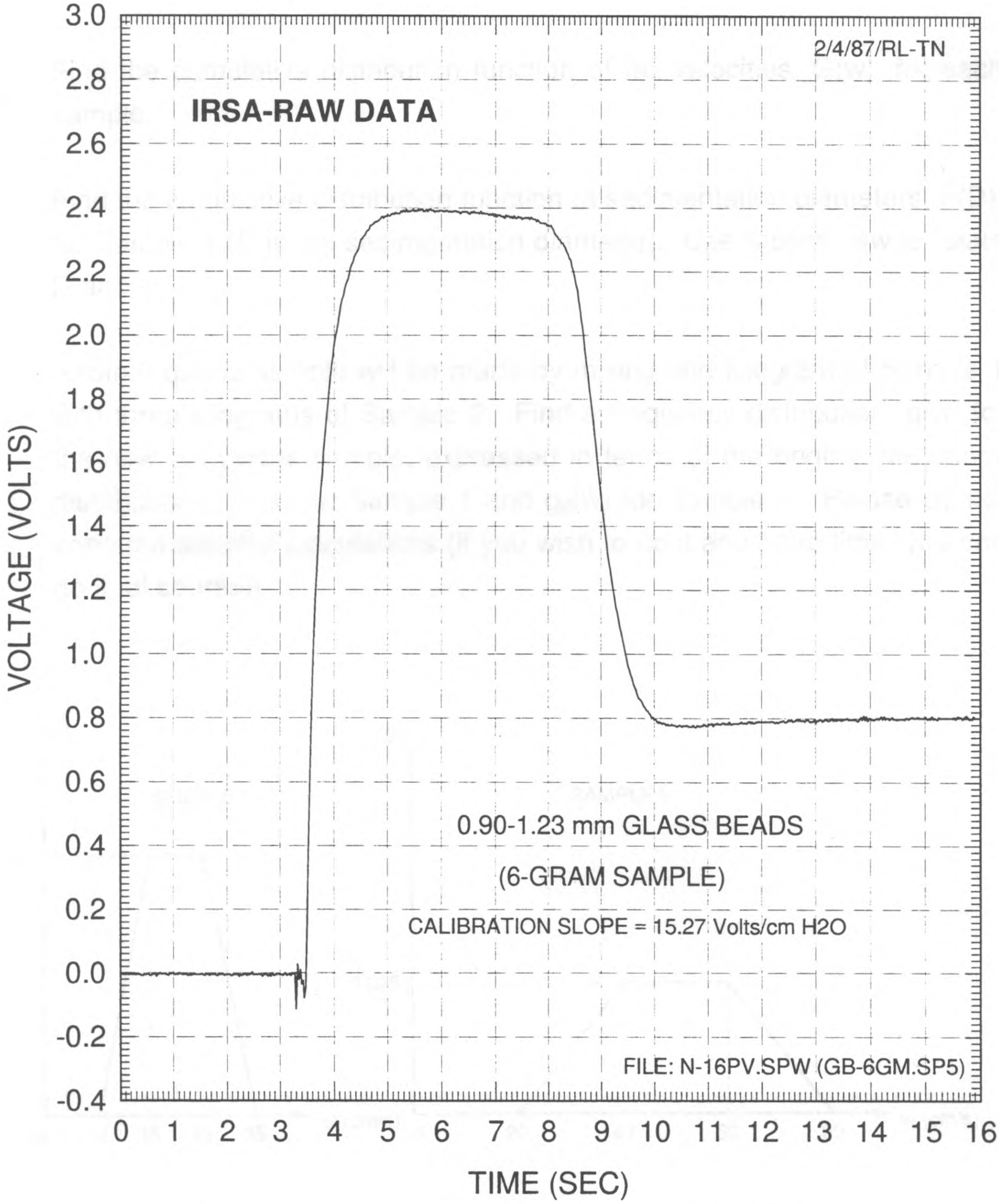


Figure N-16P Temporal changes in hydrostatic pressure

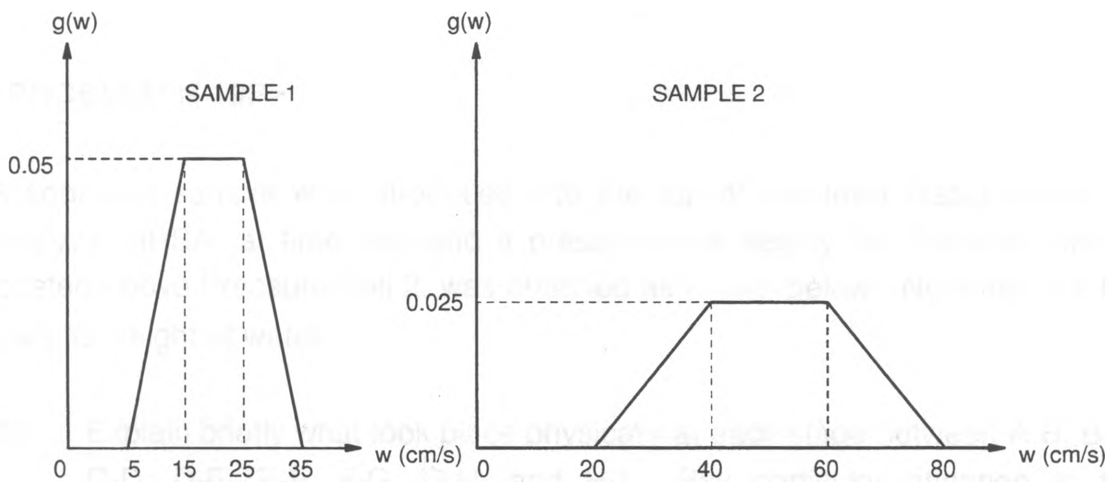
<PROBLEM N-17P>

Two different quartz-bead samples (Sample 1 and Sample 2) have frequency distributions of fall velocity, w, as shown.

(1) Find the cumulative distribution function of fall velocities, G(w), for each sample.

(2) Find the cumulative distribution function of sedimentation diameters, F(D), for Sample 1 (D is the sedimentation diameter). Use Stokes' law to relate D and w.

(3) Another quartz sample will be made by mixing one kilogram of Sample 1 with three kilograms of Sample 2. Find a frequency distribution, g(w) for the new composite sample, expressed in terms of the original frequency distributions, $g_1(w)$ for Sample 1 and $g_2(w)$ for Sample 2. Please do not carry out detailed calculations (if you wish to do it and have time, you can do it, of course!).



<PROBLEM N-18P>

A sedimentary material produced by Spherical Particles Inc. has spherical particles with a uniform diameter of D = 0.0016 ft. The fall velocity of the particles has a frequency distribution, f(w), expressed by

$$f(w) = 0 \qquad w \le 0.5 \text{ ft/s}; \qquad (1)$$

$$f(w) = (\pi/2)\sin \pi(w-0.5) \qquad 0.5 \text{ ft/s} \le w \le 1.5 \text{ ft/s; and,} \qquad (2)$$

$$f(w) = 0 \qquad w \ge 1.5 \text{ ft/s} \qquad (3)$$

Assuming that the fall velocities of the particles are given by Stokes' law,

$$w = (gD^2/18\nu)(s-1) \qquad (4)$$

find the frequency distribution of the specific gravity, g(s), of these particles in water with specific gravity equal to one and kinematic viscosity equal to 0.00001 ft^2/s.

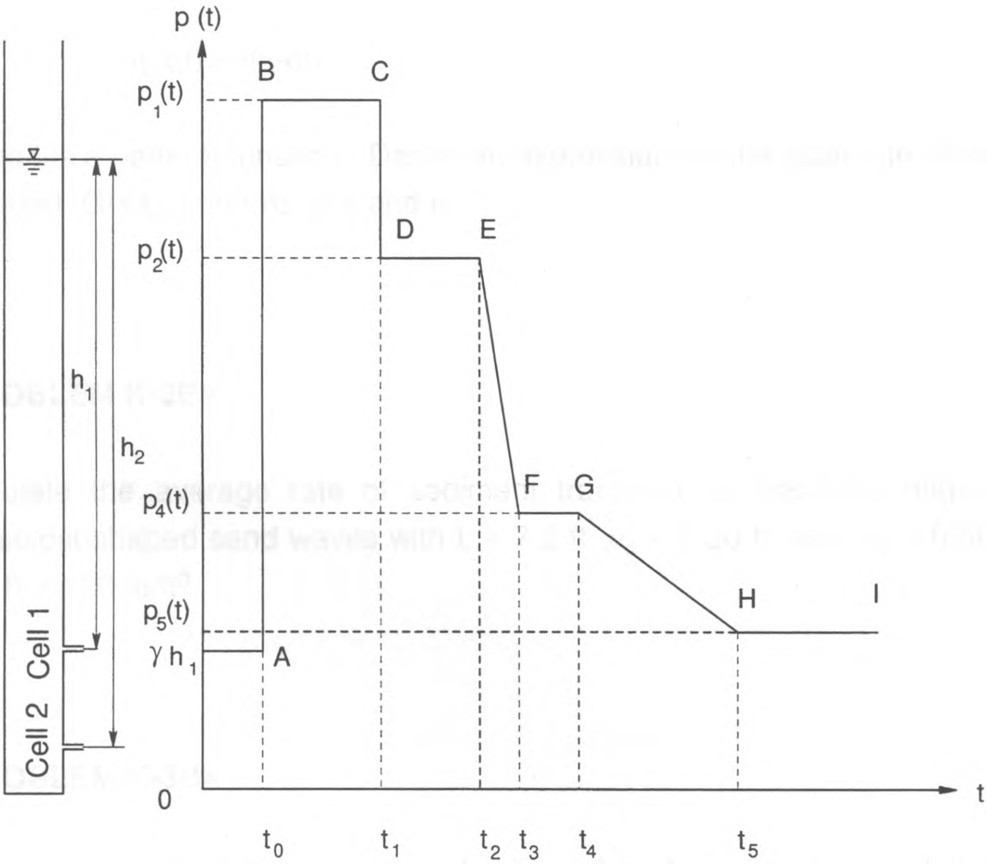
<PROBLEM N-19P>

A sediment sample was introduced into the top of the Iowa Rapid Sediment Analyzer (IRSA) at time $t=t_0$, and a pressure-time history for Pressure Cell 1, located above Pressure Cell 2, was obtained as shown below. Note that γ is the specific weight of water.

(1) Explain briefly what took place physically at each stage between A-B, B-C, C-D, D-E, E-F, F-G, G-H, and H-I. Pay particular attention to the difference in slope between E-F and G-H.

(2) From the p-t plot shown below, make a crude sketch of the expected size-frequency distribution (no detailed scaling is needed -- just a distribution shape is being sought in this case).

(3) Find the pressure-time history for Pressure Cell 2, $p_2(t)$, and relate the significant times and pressures for p_2 to those for $p_1(t)$; i.e., derive equations for the important times and pressures for the p_1-t record. Do this only for $t \leq t_3$.



<PROBLEM N-20P>

You are assigned to design a new settling tube for determining fall-velocity distributions of **fine-gravel** materials with a specific gravity of 1.5 at a room temperature of 25 °C. What is the appropriate height of the settling tube it you wish to accomplish pressure measurement of each sample within 20 seconds?

II. BED FORMS

<PROBLEM K-1B>

Consider sediment ripples or dunes that have reached their equilibrium amplitude and have a profile given by

$$\eta(x,t) = f(x-ct) \tag{1}$$

where f is a general function. Derive an expression for the local rate of sediment transport, G(x,t), in terms of η and c.

<PROBLEM K-2B>

Calculate the average rate of sediment transport by bed-form migration for sinusoidal-shaped sand waves with L = 7.0 ft, a_o = 0.30 ft, and U_b = 0.50 ft/min. Use B = 100 lb/ft^3.

<PROBLEM K-3B>

(1) Analyze the formation of ripples in a closed rectangular conduit, using a method of solution paralleling that given in class for free-surface flows. The depth of the conduit to the mean level of the bed is d, and the mean velocity is U. For a sinusoidal bed profile, the velocity potential is

$$\phi = Ux - Ua(t)\,\frac{\cosh k(y-d)}{\sinh kd}\cos k(x-U_b t) \tag{1}$$

with the y-coordinate measured positive upward from the bed.

(2) Let $d \to \infty$ to obtain the analysis for sand ripples in the desert. Show that

$$\frac{L}{h} = \frac{n\pi}{\beta}\frac{U}{U-U_c} \tag{2}$$

<PROBLEM K-4B>

Consider the formation of sediment ripples by a turbulent flow in a closed rectangular conduit, as was treated in Problem K-3B.

(1) If the mean flow depth is d, and the bed profile is given by

$$\eta = a(t) \sin k(x-ct) \tag{1}$$

derive a linearized expression for the mean velocity at any section, U(x,t).

(2) Derive an expression for $\frac{da}{dt}$. For a sediment-transport law, use

$$G(x,t) = m[U(x-\Delta, t) - U_c]^n \tag{2}$$

where $\Delta = \alpha L + \delta$, m, n, α, δ, and U_c are constants, and L is the wave length.

(3) Derive an equation for the dominant bed wave length.

<PROBLEM K-5B>

Repeat Problem K-3B using a modified form of the local transport relation given by Taizo Hayashi (*Journal of Hydraulics Division*, Proc. ASCE, 96, HY2, February, 1970),

$$G(x,t) = m\left[1+\alpha\frac{\partial\eta(x-\delta,t)}{\partial x}\right][U-U_c+\phi_x(x-\delta,0,t)]^n \tag{1}$$

where m, α, δ, and n are constants.

<PROBLEM K-6B>

A stream with a depth of 1.0 ft has antidunes that are 8.0 ft long. Estimate the water discharge, per unit width, of the stream.

<PROBLEM K-7B>

Read the following excerpt from von Kármán's paper* and derive his equation (2).

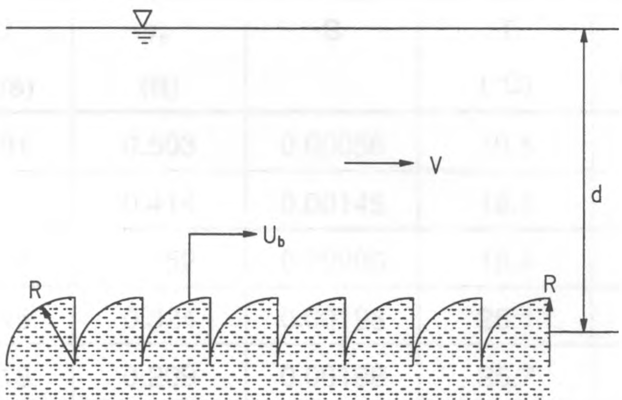
> *For qualitative discussion, the atmosphere may be considered to consist of a heavy fluid layer of relatively small width, in which all the entrained sand is distributed uniformly, with a sand-free wind stream above. Then one sees easily that the "Bernoulli effect" is opposed by the "gravity effect", since the velocity of the heavy fluid increases as it streams from the crest into the trough. There is one wave length of the ripple system for which the two effects are balanced, so that the velocity at the crest is equal to that in the trough.*
>
> *A simple analysis assuming sinusoidal shape for the surface small disturbances, leads to the following formula for the wave length:*
>
> $$\lambda = 2\pi U\sqrt{\frac{h}{g}} \qquad (2)$$
>
> *where h is the width of the heavy stream.*

---

* "Sand ripples in the deserts," *Technion Yearbook*, 6, 1947

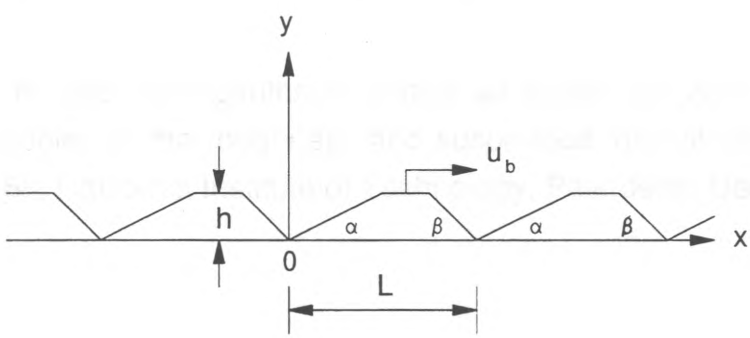
<PROBLEM K-8B>

The sand dunes on the beds of the rivers from another planet have the quarter-circle form shown in the figure below. Derive a formula, based just on the kinematics of dune movement, for the time-averaged rate of sediment transport due to migration of these dunes.



<PROBLEM N-9B>

A train of identical, trapezoidal, two-dimensional sand dunes, shown in the sketch below, moves downstream at a speed of u_b without changing form. Find the ***local*** sediment discharge, G(x), at any point x along a dune at the instant the coordinates are located as shown below.



III. FRICTION FACTORS

<PROBLEM K-1F>

Plot the following laboratory data on the Einstein-Barbarossa plot. Is the agreement with their curve satisfactory?

Run No.	U (ft/s)	r_b (ft)	S	T (°C)	Bed Configuration
3-1	0.91	0.503	0.00056	19.5	Dunes
3-4	1.14	0.414	0.00145	18.4	Dunes
3-2	1.35	0.352	0.00206	18.4	Dunes
3-7	1.45	0.324	0.00198	25.1	Dunes
3-6a	2.13	0.209	0.00198	25.7	Flat
3-6	2.15	0.209	0.00207	25.2	Flat
3-6b	2.21	0.205	0.00221	25.4	Flat

q	=	0.50 cfs/ft for all runs;
Flume width	=	33.5 in.;
Flume length	=	60 ft;
D_{35}	=	0.125 mm;
D_{65}	=	0.162 mm; and,
$D_g = D_{50}$	=	0.142 mm

To determine R', use the logarithmic relation as plotted by Vanoni and Brooks ("Laboratory studies of the roughness and suspended load of alluvial streams'" Report No. E-68, California Institute of Technology, Pasadena, December 1957).

<PROBLEM K-2F>

The lengths and heights of the dunes formed by the runs summarized in Problem K-1F are as follows:

Run No.	Average Dune Length L (ft)	Average Dune Height h (ft)
3-1	0.43	0.049
3-4	0.46	0.064
3-2	0.48	0.070
3-7	0.42	0.066

(1) Find f' and f" for each run, using D_g as the equivalent sand-grain roughness.

(2) Assuming the bed features to be two-dimensional, what is the drag coefficient, C_D, based on the average velocity of the flows?

(3) Find f' for the flat-bed flows (Runs 3-6a, 3-6, and 3-6b) using a pipe-friction diagram and Lovera's curve ("Hydraulic relations for alluvial streams," *Journal of Hydraulic Division*, Proc., ASCE, 97, HY1, January 1971).

<PROBLEM K-3F>

One of the central equations in Engelund's analysis is

$$U = \sqrt{gR'S}\left[6.0 + 2.5\ln\frac{R'}{k_s}\right] \tag{1}$$

where R' is determined from a plot of experimental data. If typical scatter on R' is given by

$$\frac{dR'}{R'} = 0.2 \qquad (2)$$

Find the corresponding error in velocity dU/U.

<PROBLEM K-4F>

The Rio Grande at Bernalillo, New Mexico, has the following characteristics:

Depth range:	d	=	0.8 - 4.5 ft;
Velocity range:	U	=	1.5 - 8 ft/s;
Width:	W	=	350 ft;
Slope:	S	=	0.00085; and,
Temperature	T	=	60 °F on average

Bed-material properties:

D_g	=	0.29 mm;
σ_g	=	1.21;
D'_{35}	=	0.24 mm;
D'_{50}	=	0.28 mm; and,
D'_{65}	=	0.35 mm

Prepare a stage (depth)-velocity curve for the Rio Grande at this site, using the Einstein-Barbarossa procedure, Engelund's method, and Alam-Lovera-Kennedy method. Plot the results of all three calculations on a single graph, and discuss your results.

<PROBLEM K-5F>

A river is 500 ft wide, 10 ft deep, and has a slope of one foot per mile and side slopes of 1 vertical to 1-1/2 horizontal. The bed sediment has a geometric mean size of 0.5 mm and a geometric standard deviation of 1.5. The banks are covered with vegetation and have a Manning roughness coefficient of n = 0.030. Find the roughness coefficient of the bed and the mean velocity using the Einstein-Barbarossa method and the sidewall correction method described by Vanoni and Brooks.

<PROBLEM K-6F>

A trapezoidal irrigation canal has a bottom width of 4 m, side slopes of 1 vertical to 1-1/2 horizontal and a slope of 0.00030. The banks which are heavily vegetated have a Manning roughness of n = 0.030, while n for the bed is 0.020. Calculate the discharge and the mean shear stresses exerted on the bed and the banks, using the sidewall correction method for a depth of 1.2 m.

<PROBLEM K-7F>

The following data were obtained from an experiment in a rectangular flume with smooth walls:

W	=	33.5 in. (flume width);
V	=	1.35 ft/s (mean flow velocity);
d	=	0.37 ft (mean flow depth);
S	=	0.00206 (slope of energy grade line);
T	=	65 °F (water temperature);
ν	=	1.13×10^{-5} ft^2/s (kinematic viscosity of water); and,
D_g	=	0.142 mm (geometric mean diameter)

(1) Find τ_b, the shear stress for the bed section.

<PROBLEM N-8F>

A trapezoidal irrigation canal in Farms of Texas in Alvin, Texas has a bottom width of 6 m, side slopes of 1 vertical to 2 horizontal, a bed slope of 0.0005, and a median bed-material size of 0.5 mm. The banks, which are heavily vegetated and infested with water snakes (particularly water moccasins), have a Manning's roughness of 0.050, while the n value for the bed is 0.025.

(1) Estimate, using the sidewall correction method, the water discharge and the mean shear stresses exerted on the bed and banks when the flow depth is 2.0 m.

(2) What kind of bed forms would you expect under these flow conditions?

IV. INITIATION OF MOTION

<PROBLEM K-1I>

A laboratory experiment at Colorado State University yielded the following data for an experiment with incipient sediment motion:

> Bed material: sand with D_g = 0.19 mm = 0.000624 ft;
> Transport fluid: water with $\nu = 1.11 \times 10^{-5}$ ft^2/sec;
> Energy grade line slope = 0.00005; and,
> Flow depth = 0.96 ft.

Is this experiment consistent with Shields' relation for incipient motion?

<PROBLEM V-2I>

From a Shields' graph, calculate the critical shear stress for a quartz sand with mean size of 0.25 mm. Assume a water temperature of 20 °C.

<PROBLEM V-3I>

C.M. White * obtained the following expression for the critical shear stress, τ_c, for bed material,

$$\tau_c = 0.18(\rho_s - \rho)gD\tan\alpha \tag{1}$$

in which D is the grain size; α is the angle of repose of the material; and ρ_s and ρ are the mass densities of the sediment and fluid, respectively. How does this agree with the corresponding results of Shields? Explain any differences.

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* "The equilibrium of grains on the bed of a stream," *Proc. Royal Society*, Series A, 174, 958, February 1940.

<PROBLEM V-4I>

Derive a relation between critical shear stress for initiation of motion of bed sediment and mean flow velocity.

<PROBLEM V-5I>

What is the critical velocity of a flow 3 ft deep over a flattened bed of sand with a medium size of 0.3 mm?

<PROBLEM V-6I>

A wide stream is 4 ft deep; has a slope of 0.00015; and the sand in the bed has a medium size of 0.5 mm.

(1) Will the sediment move?

(2) Will any sediment be in suspension?

<PROBLEM V-7I>

The banks of the Los Angeles River near Compton are revetted with quarry stone with an average weight of 45 kg per stone. The bottom width of the channel is 150 m; the bank slope is 1 vertical to 2 horizontal; and the channel slope is 0.00080. At design discharge, the water depth is 4.3 m. Is the revetment stable? The stone has a specific weight of 2.5 tons/m^3.

<PROBLEM V-8I>

Assume that you are given the job of designing a channel with a loose boundary so that none of the sediment forming the boundary will move. Would you use the Shields curve to calculate the safe value of the boundary shear stress? If not, how would you proceed?

<PROBLEM N-9I>

On 5 May 1988 the IIHR river crew collected several bed-material samples from the Missouri River using U.S.G.S. bed-material sampler BM-50. The result of the sieve analysis done for one of the samples is shown below. This sample was taken 60 ft away from the circulating-water pump intake of the Omaha Public Power District, Nebraska City Station located along the concave bank of the Missouri River at River Mile (RM) 556.2 near Nebraska City, Nebraska. The local mean flow velocity measured was 4.06 ft/s; local depth-averaged suspended-sediment concentration was 142 ppm; and local flow depth was 23 ft.

(1) Plot the size frequency curve and determine D_{16}, D_{50}, D_{84}, D_g, and σ_g.

(2) Under the given conditions, what grain size material was in motion?

(4) If the mean velocity increases by 20%, what percentage of bed material would move as bed load?

Sieve Opening (mm)	Amount Retained (grams)
16.000	0.00
8.000	19.87
4.000	62.59
2.000	47.91
1.000	46.47
0.500	45.68
0.250	6.56
0.125	2.40
0.074	0.76
PAN	0.32

<PROBLEM N-10I>

An IIHR field crew headed by Tatsuaki Nakato obtained the following data on 13 April 1988 near River Mile 556.3 (RM: mileage measured from its mouth in St. Louis) of the Missouri River immediately upstream from Omaha Public Power District Nebraska City Station's circulating-water pump intake:

Water temperature	T	=	46 °F;
Mean flow velocity,	U	=	4.73 ft/s;
Mean flow depth,	d	=	15.2 ft; and,
Geometric mean size of bed material,	D_g	=	0.75 mm

(1) Find Darcy-Weisbach's friction factor and estimate the energy slope under these conditions. Please make sure that you clearly identify the procedure you have employed.

(2) What kind of bed forms do you expect under these conditions? Why?

(3) Estimate, using the Shields curve, the largest bed-material size that can be moved under these conditions.

V. SUSPENDED-LOAD DISCHARGE

<PROBLEM K-1S>

Using a circular cylindrical control volume, derive the general convection-diffusion equation for suspended sediment in cylindrical coordinates, including the components of the fall velocity in the r, z, and θ directions.

<PROBLEM K-2S>

In a steady, uniform flow in a wide open channel, the velocity distribution is found to be given by

$$\frac{u(y)}{u(d)} = (y / d)^{1/n} \qquad (1)$$

where u(d) is the maximum (surface) velocity at y/d = 1. Derive an equation for the distribution of suspended-sediment concentration. You may assume 1/n <<1 if necessary when integrating the equation.

<PROBLEM V-3S>

A wide stream 4 ft deep has a slope of 0.00015 and the sand on the bed has an average size of 0.5 mm.

(1) Will there be appreciable bed material in suspension?

(1) Will the flow appear turbid to an observer on the bank? Give reasons for your answer. The water temperature is 60 °F.

<PROBLEM V-4S>

Two suspended-load samples were taken at a vertical of a stream 4 m deep. The first sample taken at a depth of 1 m below the surface had a concentration of 0.05 grams per liter of a given fraction of sediment. The other sample taken at a depth of 3 m had a concentration of 0.5 grams per liter of the same size fraction. What is the mid-depth concentration of the size fraction?

<PROBLEM V-5S>

Given the following velocity and sediment profiles at the centerline of a flume 33 inches wide set on a slope of 0.0025 (RUN 43 Sta 45 Center).

(1) Plot the velocity profile on a semi-logarithmic graph and determine κ.

(2) Plot the log C versus log (d-y)/y and determine z.

(3) Calculate the rate of suspended-sediment transport per unit width.

Note that the flow depth d = 0.298 ft; the size of suspended-load particles = 0.106 mm; and settling velocity of suspended-load particles = 0.0301 ft/s.

y (ft)	u (ft/s)	C (grams/liter)
0.01	2.03	32.080
0.02	2.15	24.670
0.05	2.54	15.410
0.07	2.73	11.960
0.10	2.98	8.980
0.15	3.31	5.360
0.20	3.53	2.960
0.25	3.69	1.300
0.27	3.75	0.866

<PROBLEM K-6S>

The table below gives the distributions of local time-averaged mean velocity and suspended-sand concentration for the fraction of the material passing the 0.104 mm sieve and retained on the 0.074 mm sieve at vertical C-3 on the Missouri River at Omaha on 4 November 1952. On this day, the slope of the stream was 0.000120; the depth was 7.8 ft; the width was 800 ft; the water temperature was 7 °C; and the flow was approximately uniform.

Distance up from Bottom y (ft)	Local Velocity u (ft/s)	Suspended-Sediment Concentration in Size Fraction 0.074 - 0.104 mm C (grams/liter)
0.7	4.30	0.411
0.9	4.50	0.380
1.2	4.64	0.305
1.4	4.77	0.299
1.7	4.83	0.277
2.2	5.12	0.238
2.7	5.30	0.217
2.9	5.40	------
3.2	5.42	0.196
3.4	5.42	------
3.7	5.50	0.184
4.2	5.60	------
4.8	5.60	0.148
5.8	5.70	0.130
6.8	5.95	------
7.8	----	------

(1) Plot the velocity profile on semi-logarithmic paper (u vs. log y) and concentration profile on log-log paper (C vs. (d-y)/y). Draw straight lines giving the best fit to the points.

(2) Compute from the data given and your graphs the following quantities:

- u_*: shear velocity;
- U: mean velocity;
- κ: von Kármán constant;
- f: Darcy-Weisbach friction factor; and,
- z: exponent of suspended-load distribution equation

(3) Compute the rate of transport of this particular size fraction of sand in pounds per second per foot by graphical integration of the product (Cu).

(4) Prepare a "bar graph" showing the fraction of the total suspended load for this size fraction carried in each 10% of the depth.

(5) Obtain the transport rate of this size fraction from Brook's nomogram.

(6) Using tabulated values of the Einstein integrals, obtain the transport rate using the other two values of the lower lines of the integral which were discussed in class.

(7) Using the value of u_* obtained from the Einstein-Barbarossa procedure for this unit discharge, find the suspended load of this size fraction using Brook's nomogram. Note that D_g = 0.20 mm and σ_g = 1.20 for the bed material.

(8) Plot the concentration distribution in the format of the relation derived from the power-law velocity distribution.

(9) Calculate the suspended-sediment discharge for this size fraction given by the power-law velocity-distribution formulation.

(10) If D_{50} = 0.25 mm, find the total suspended-load discharge using the power-law formulation.

<PROBLEM K-7S>

A wide stream has a depth of 10 ft and a slope of 0.003; and its bed material is well sorted sand with a mean size of 0.50 mm.

(1) Estimate the percent increase in the suspended-load discharge when the temperature of the water decreases from 60 °F to 32 °F.

(2) If the bed of this stream were composed of 0.25 mm sand, would the percent increase in the suspended-load discharge with the above decrease in temperature be greater or less? Give the basis for your answers. Assume that the bed-load discharge, the concentration near the bed, and the mean velocity do not vary with temperature.

<PROBLEM V-8S>

The velocity distribution of sediment-transporting rivers on a newly discovered planet in another galaxy is found to be given by

$$v = \left(\frac{u_*^2}{w}\right)(\beta m)\left(\frac{y}{d}\right) \qquad (1)$$

where $m \geq 0$ is a parameter and the other notation has the same definitions as on Earth.

(1) Find the mean velocity of the stream, and the elevation at which it occurs.

(2) Derive an expression for the Darcy-Weisbach friction factor of the flow.

(3) Derive the equation expressing the vertical distribution of suspended-sediment concentration. Make the same assumptions as are employed in the Ippen-Vanoni analysis for Earth rivers.

(2) Derive an equation expressing the discharge of suspended sediment in these extraterrestrial streams.

<PROBLEM K-9S>

A certain river has the following characteristics:

Bed material: silica sand

Bed material diameters:

$D_{35} = 0.25$ mm = 8.20×10^{-4} ft;

$D_{50} = 0.30$ mm = 9.84×10^{-4} ft;

$D_{65} = 0.35$ mm = 11.48×10^{-4} ft;

Channel width:	$B = 250$ ft;
Mean velocity:	$V = 5.25$ ft/s;
Mean depth:	$d = 2.00$ ft;
Water temperature:	$T = 73$ °F; and,
Kinematic viscosity:	$\nu = 1.05 \times 10^{-5}$ ft²/s

(1) Determine the median fall velocity of the bed material. Assume a shape factor of unity.

(2) Determine the geometric standard deviation, σ_g, of the bed-material size distribution.

(3) Find the friction factor and slope of the stream. Use the Alam-Lovera procedure, and D_{50} as the characteristic bed-material size.

(4) Use the Shields curve to estimate the largest bed-material size that can be moved by this flow.

(5) For material in the size range $0.25 \text{ mm} \leq D \leq 0.35$ mm, the *volumetric* concentration of suspended sediment at a distance $y = 5.55 \times 10^{-3}$ ft above the bed is found to be 0.0766. Compute the mid-depth concentration of material in this size range. Use the Rouse-Ippen-Vanoni relation, and assume $\kappa = 0.3$ and $\beta = 1.0$.

(6) Determine the sediment discharge of material in the size range of Part (5) above. Use the Brooks nomogram.

(7) This flow is observed to form antidunes. Compute the wave length of the antidunes.

(8) Compute the water discharge of this flow.

<PROBLEM K-10S>

A wide stream with depth d, and slope S has a power-law velocity distribution,

$$\left(\frac{n+1}{n}\right)\left(\frac{u}{V}\right)=\left(\frac{y}{d}\right)^{\frac{1}{n}} \tag{1}$$

where V = mean flow velocity. The turbulent exchange coefficient for suspended sediment may be assumed to be constant with an effective value of

$$\varepsilon_S = mu_*d \tag{2}$$

where m = constant. Derive an expression for the suspended-sediment discharge and discuss your choice of lower limit of integration.

Note:
$$\int x^r e^{ax}dx = \frac{x^r e^{ax}}{a} - \frac{r}{a}\int x^{r-1}e^{ax}dx \tag{3}$$

You may leave your answer in integral form if you wish.

<PROBLEM N-11S>

Consider a simple depth-integrating suspended-sediment sampler in which no air escapes from the container and the air in the container will be compressed by the changing hydrostatic head so that the reduction in air volume is balanced by the inflowing water.

(1) Show that the lowering rate of the sampler, R_L, at any instance is given by

$$R_L = \frac{dD}{dt} = \frac{Av(h_1+D)^2}{h_1V_1} \tag{1}$$

where

D = vertical depth measured from water surface; t = time; A = entrance cross-section area of the intake nozzle; v = local streamwise flow velocity at vertical depth, D; h_1 = absolute pressure head at water surface = 34 ft; and V_1 = volume of the sampler container.

(2) Assuming a power-law velocity distribution,

$$\frac{v}{V} = \left(\frac{n+1}{n}\right)\left(\frac{D_s - D}{D_s}\right)^{\frac{1}{n}} \tag{2}$$

where

V = depth-averaged mean velocity; D_s = flow depth; and 1/n=exponent (assume n=8 in this case).

rearrange equation (1) to

$$\frac{R_L}{V} = \frac{Ar(h_1 + D_s d)^2}{h_1 V_1}; \quad \left(r = \frac{v}{V}\right) \tag{3}$$

and plot four curves showing relationships between R_L and D/D_s (=d) for D_s = 10 ft, 20 ft, 30 ft, and 40 ft. Plot the velocity profile in the same figure (horizontal axis = R_L/V; and vertical axis = vertically decreasing d). Assume that the container intake nozzle is 3/16 in. in diameter and the sample container is a one-pint milk bottle ($V_1 = 1.671 \times 10^{-2}$ ft^3).

<PROBLEM N-12S>

Answer the following questions related to Fredsϕe's paper (*Journal of Fluid Mechanics,* 1978, Vol. 94, Part 4, pp.609-624).

(1) Show that the longitudinal component of the suspended-load discharge, q_{s1}, is given by

$$q_{s1} = \int_h^D (U + u_1)(c_0 + \tilde{c})dx_2 = -hc_{b0}U + q_{s0} + EUDG \tag{1}$$

in which

$$G = \int_0^1 \left[\phi + \frac{u_{10}}{U} c_{b0} \exp\left(-\frac{wD}{\varepsilon} \frac{x_2}{D} \right) \right] d\left(\frac{x_2}{D} \right); \tag{2}$$

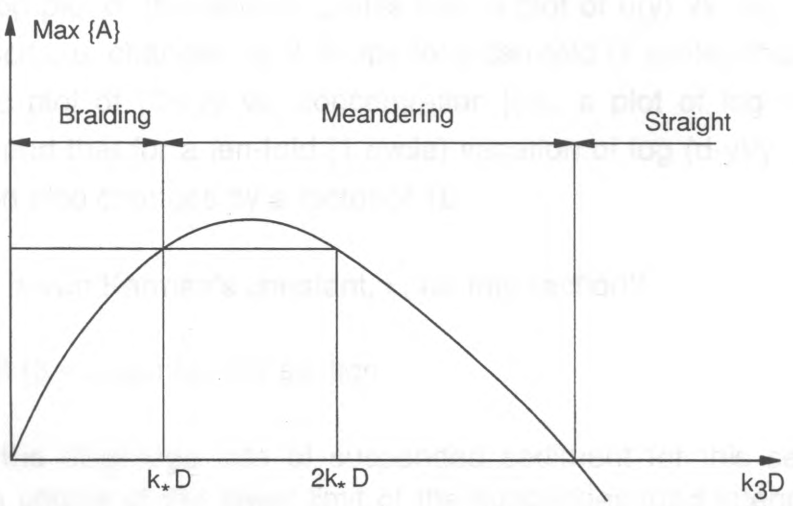
$$\tilde{c} = \phi E; \quad u_1 = u_{10}E; \quad E = \cos(k_3x_3)\exp[ik_1(x_1 - at)]; \tag{3}$$

$$c_0 = c_{b0}\exp[-\frac{wx_2}{\varepsilon}]; \tag{4}$$

and q_{s0} = suspended-load discharge for an undisturbed flow.

(2) According to Fredsϕe's 1978 analysis, the maximum amplification factor (longitudinal amplification) **A** is related to k_3D, as shown in the following figure. Explain from the standpoint of physics why a channel starts braiding when $k_3 \le k_*$ and starts meandering when $k_3 \ge k_*$.

[Hint]: Consider the boundary conditions for $u_3 = u_{30}\tan(k_3x_3)E$



VI. TOTAL-LOAD DISCHARGE

<PROBLEM K-1T>

The following data were obtained from the Colorado River near Taylor's Ferry on 24 July 1956:

River width = 354 ft;
Average depth = 10.66 ft;
Average velocity = 3.23 ft;
Water temperature = 80 °F = 27 °C;
Energy slope = 0.000227;
Kinematic viscosity = 0.93×10^{-5} ft^2/s;
Mean size of suspended sediment = 0.148 mm;
Suspended-sediment concentration at mid-depth: C_{md} = 80 ppm;
Size distribution of bed material;

D'_{35} = 0.290 mm = 0.950×10^{-3} ft;
D'_{50} = 0.332 mm = 1.090×10^{-3} ft; and,
D'_{65} = 0.379 mm = 1.240×10^{-3} ft

On a semi-log plot of the velocity profile [i.e., a plot of u(y) vs. log y], it is found that the velocity, u, changes by 2.30 fps for a ten-fold (1 cycle) change in y. On a logarithmic plot of (d-y)/y vs. concentration [i.e., a plot of log (d-y)/y vs. log C(y)], it is found that for a ten-fold (1 cycle) variation of log (d-y)/y, the sediment concentration also changes by a factor of 10.

(1) What is von Kármán's constant, κ, for this section?

(2) Find β ($\beta = \varepsilon_s/\varepsilon_m$) for this section.

(3) Find the discharge rate of suspended sediment for this section. State which choice of the lower limit of the suspended-load integrals you have chosen.

(4) Briefly tell how you would calculate the bed-load discharge for this section.

(5) u_*' for this section is given as 0.108 fps. Find the velocity predicted by the Einstein-Barbarossa method.

(6) Show qualitatively the form of the stage-discharge (R vs. Q) relation to be at this section. Briefly discuss the significance of any important characteristics of the curve.

(7) Estimate what the bed form was for this flow condition. Give the basis for your estimate.

(8) Why is the suspended material finer than the bed material? Is this normal?

<PROBLEM K-2T>

The Colorado River at Taylor's Ferry, California, has the following characteristics:

D'_{35} = 0.287 mm = 0.942 x 10^{-3} ft;
D'_{50} = 0.330 mm = 1.083 x 10^{-3} ft;
D'_{65} = 0.378 mm = 1.240 x 10^{-3} ft;
D_g = 0.320 mm = 1.050 x 10^{-3} ft;
σ_g = 1.44;
Slope = 0.000217;
Width = 350 ft; and,
Water temperature = 60 °F

The bed material may be divided into size fraction as follows:

p_i (%)	D_i mm (ft)	w_i cm/s (ft/s)
20.8	0.177 (0.00058)	1.9 (0.063)
69.6	0.354 (0.00116)	4.8 (0.158)
9.6	0.707 (0.00232)	9.6 (0.314)

(1) Use the Lovera-Alam method to calculate a rating curve for this section. Then use the Einstein Bed-Load Function procedure to calculate q_s and q_b for this channels for a unit discharge q = 20 cfs/ft. The measured *total load* varies from 0.12 to 0.54 lb/sec/ft at this section.

(2) Calculate q_b from DuBoys' formula.

(3) Calculate the total load using Toffaleti's approach.

(4) Determine the sediment load predicted by Meyer-Peter and Mueller's formula.

<PROBLEM K-3T>

The following data were obtained from the Rio Grande at Section A-2, Bernalillo, New Mexico, on 2 June 1953 [Refs., U.S.G.S. Professional Paper 462-B (Nordin and Demster, 1963); and 462-F (Nordin and Beverage, 1965)]:

Size of bed material:

D'_{35} = 0.27 mm = 0.885 x 10^{-3} ft;
D'_{50} = 0.32 mm = 1.05 x 10^{-3} ft;
D'_{65} = 0.37 mm = 1.21 x 10^{-3} ft;

River width = 270 ft;
Mean depth = 2.56 ft;
Channel slope = 0.00083;
Water temperature = 71 °F; and,
Kinematic viscosity = 1.04 x 10^{-5}

(1) For the suspended material in the size range 0.25 mm ≤ D ≤ 0.50 mm, the logarithmic plot of (d-y)/y vs. concentration shows that for a ten-fold (one cycle) change in (d-y)/y corresponds to a change of exactly one-third of a log cycle in C. A plot of velocity, u(y), vs. log y, where y is the distance

from the bottom, yields a straight line with the velocity changing 1.20 fps for a ten-fold change in y. Find Kármán's constant, κ, and also $\beta=\varepsilon_s/\varepsilon_m$.

(2) It is known that $\Psi' = 1.59$ for the flow at this section. Find the mean velocity, U.

(3) The mid-depth concentration of this size fraction is 62 ppm. Find the suspended-sediment discharge. State clearly what lower limit of the suspended-load integral you used.

(4) By extrapolating the suspended-load formula down to y = 2D, find the concentration in the bed layer.

(5) What is the bed-load transport rate?

(6) Outline concisely the steps you would follow in obtaining i_b for the size fraction, given that i_B=0.4 and that the bed material diameters are log-normally distributed. *Do not carry out the calculation!*

(7) What was the bed configuration? State the reasons for your conclusion.

(8) During another flow, antidunes 10 ft long were observed at this location. The flow depth was 3.7 ft. Estimate the unit discharge for this flow condition.

<PROBLEM K-4T>

The following data were obtained for the Rio Grande at Section D on 19 May 1954 [Ref.: U.S.G.S. Professional Paper 462-B (Nordin and Dempster, 1963) and U.S.G.S. Professional Paper 462-F (Nordin and Beverage, 1965)].

D'_{35} = 0.290 mm = 0.820×10^{-3} ft;
D'_{50} = 0.332 mm = 1.018×10^{-3} ft;
D'_{65} = 0.379 mm = 1.248×10^{-3} ft;

River width:	=	401 ft;
Mean depth:	=	1.50 ft;
Slope of energy grade line:	=	0.000864;
Water temperature:	=	73 °F; and,
Kinematic viscosity:	=	1.05×10^{-5} ft^2/s

(1) A plot of concentration vs. (d-y)/y for the suspended material in the size range 0.25 mm < D< 0.50 mm yields a straight line which for a ten-fold (1 log cycle) change in (d-y)/y, C changes from 2.5 ppm to 14 ppm. A plot of u(y) vs. log y, where y is the upward distance from the bottom, yields a straight line with the velocity changing 1.0 fps for each one log-cycle change in y. Find von Kármán's constant κ and also $\beta = \varepsilon_s/\varepsilon_m$.

(2) For this flow, $\Psi' = 3.80$. Find the mean velocity, U, and compare with the measured velocity, $U_m = 2.35$ fps.

(3) The mid-depth concentration of this size fraction is 14 ppm. Find the suspended-sediment discharge. State clearly what lower limit you used in the suspended-load integral.

(4) Extrapolate the suspended-load formula down to y = 2D to find the bed-layer concentration.

(5) Find the transport rate of the bed load for this size fraction (given range of D).

(6) Estimate what the bed configuration was for this flow. State the reasons and show supporting data for you conclusion.

Note: $$i_B q_B = \phi_* i_b \rho_s g^{3/2} D_i^{3/2} (s_s - 1)^{1/2} \quad (1)$$

$$q_{si} = 11.6 u_*' C_a a [2.303 \log_{10}(\frac{30.2d}{\Delta}) I_1 + I_2] \quad (2)$$

(in case you want to use these formulas)

<PROBLEM V-5T>

The data in the following table were observed in the Missouri River at Omaha on two days during which the discharge was 27,000 cfs;. the slope was 0.000145; and the bed-sediment size properties were $D'_{35} = 0.17$ mm, $D'_{50} = 0.20$ mm, $D'_{65} = 0.23$ mm.

Date	Water Temperature (°F)	Mean Depth (ft)	Mean Velocity (ft/s)	Suspended-Load Sediment Concentration (ppm)	
				Sediment Coarser than 0.074 mm	Size fraction of 0.125 mm mean size
20 Aug 1968	75	11.1	4.28	220	90
15 Nov 1968	40	9.7	5.15	560	200

(1) Explain those observed temperature effects that you can in terms of fluid mechanics.

(2) Calculate the concentration of suspended sediment of mean size 0.125 mm at the level $y/d = 0.05$ for 20 August and 15 November 1968.

(3) On which of the two days was the bed-load discharge of 0.125 mm sediment the greater?

<PROBLEM I-6T>

Two wide canals of the same width are to be built in the same alluvial deposit (mean size and specific gravity of bed sediment: $D'_{50} = 8.2 \times 10^{-4}$ ft and S.G. = 2.65). The second canal is to have a slope of 20% greater than the first ($S_1 = 0.002$). To retain the same sediment-transport rate in both canals, when the depth of the first is fixed at 20 ft, what must be the depth of the second canal? Utilize the following expression:

$$g_s = \frac{10qS}{\gamma}\left[\frac{\tau_0 - \tau_c}{(s_s - 1)^2 D_{50}}\right] \quad \text{>> Shields' transport equation} \qquad (1)$$

$$q = \frac{1.49}{n} y^{\frac{5}{3}} S^{\frac{1}{2}} \quad \text{>> Discharge equation} \qquad (2)$$

$$\frac{\tau_c}{D_{50}\gamma(s_s - 1)} = \text{const} = 0.06 \quad \text{>> Critical shear-stress equation} \qquad (3)$$

<PROBLEM I-7T>

Assume the classical DuBoys equation for bed-load sediment transport to hold:

$$g_s = \Psi_D \tau_0 (\tau_0 - \tau_c) \qquad (1)$$

in which the critical shear stress τ_c indicates beginning of sediment transport. The sediment parameter is $\Psi_D = 0.170 / D_{50}^{3/4}$ with D_{50} = mean sediment diameter in mm. In laboratory tests in a rectangular channel the critical shear stress was reached at a given slope for a sediment diameter of D_{50} = 1 mm at a depth of flow of y_0 = 3 in., and was found to be τ_c = 0.033 psf. Assume the value of Manning's n to be 0.013 at this condition of flow. What sediment transport rate is to be expected, if the discharge in the flume is doubled? What conditions might jeopardize this forecast?

<PROBLEM I-8T>

The following two equations are assumed to apply to a certain length of a river channel:

The discharge equation: $$q = \frac{1.49}{n} B y_m^{5/3} S^{1/2} \qquad (1)$$

The sediment-transport equation: $$g_s = \left[\frac{0.17}{(D_{50})^{3/4}}\right]\gamma^2 B(y_m)^2 S^2 \qquad (2)$$

It has been proposed to deepen the channel for navigation to twice the mean depth of y_m. It is desirable to maintain the existing sediment-transport rate by proper adjustment of the channel geometry in width and length.

(1) What measures are implied by the above equations?

(2) Are these measures feasible? State the reasons for your conclusion.

<PROBLEM V-9T>

Two wide streams with water temperature of 60 °F flow over beds of identical sands with means velocities of 3 ft/s. One stream is 3 ft deep and the other is 6 ft deep. The size properties of the bed sediments are D_{50} = 0.30 mm, D_{35} = 0.25 mm, and D_{65} = 0.35 mm.

<PROBLEM V-10T>

An engineer needs to know the sediment discharge of a river at a water discharge equal to the mean annual discharge with an error not to exceed 20 percent. How would you advise him to get the sediment discharge?

<PROBLEM K-11T>

On 24 May 1960 the following data were obtained at Section A-2 of the Rio Grande (U.S.G.S. *Water Supply Paper* 1498 H):

V = 2.71 ft/s;
S = 0.00083; and
ν = 1.13×10^{-5} ft²/s

You may assume, *for the purposes of this problem only*, that the bed material is uniform with $D_{35} = D_{50} = D_{65} = 0.29$ mm $= 9.51 \times 10^{-4}$ ft and $x = 1$.

(1) Calculate the depth using the Einstein-Barbarossa procedure. *Note that this does not require a trial-and-error procedure.*

(2) Calculate the bed-load discharge using the Einstein bed-load function.

(3) Calculate the suspended-load discharge using the Einstein procedure.

<PROBLEM K-12T>

Design an irrigation channel to convey a water discharge of 1,000 cfs and a sediment discharge of 500 tons/day. The slope of the channel is 0.00012. The transported material has $D_g = 0.25$ mm, and $\sigma_g = 1.4$. The water temperature is 70 °F. The banks are a fairly stiff clay. Clearly list all assumptions you make.

<PROBLEM K-13T>

An alternate design for the canal of Problem K-12T involves a "de-silting" works, which would reduce the sediment discharge in the canal to 30% of its original value. Design a canal for these conditions.

<PROBLEM K-14T>

For the data of Problem K-4T, calculate the total-load sediment discharge using:

(1) Toffaleti's procedure

(2) Karim's procedure

<PROBLEM K-15T>

For the data given in Problem K-4T, compute the total sediment discharge for U = 2.35 fps using Yang's equations (ASCE, C.T. Yang, 1973 and 1979).

<PROBLEM K-16T>

A measured flow (No. 8 of Brownlie's compilation) in the Rio Grande Conveyance Channel (a man-made canal in New Mexico) had the following properties:

Q	=	39.08 m^3/s;	ν	=	1.032×10^{-6} m^2/s;
d	=	1.51 m;	D_{50}	=	0.028 mm (bed material);
B	=	22.86 m (width);	σ_g	=	1.49 (bed material);
g	=	9.81 m^2/s;	Bed material = quartz sand;		

Suspended-load discharge = 0.0212 m^3/s; and

Suspended-load sediment volumetric concentration = 5.43×10^{-4}

(1) Determine the median fall velocity of the sediment. Assume that the particles are spherical.

(2) What is $D_{84.1}$ of the bed material?

(3) Determine the mid-depth concentration using the Brooks nomogram. Assume that the size of the suspended-load sediment is the same as the size of the bed material.

(4) Calculate the sediment concentration at the elevation $y = y_b = D50(u_*/u_{*c})$ [Karim's bed-layer thickness]. Use the Rouse-Ippen-Vanoni formulation.

(5) Estimate the bed-load discharge using Karim's procedure, and the results of Parts (3) and (4) above.

(6) Use an "inverted Einstein procedure" to obtain another estimate of the bed-load discharge from

$$q_B = 11.6\ C_a\ u_*'\ a \tag{1}$$

Do *NOT* obtain Φ_*. Treat the sediment as uniform (i.e., $i_B = 1$).

(7) For this channel, D_{35} = 0.238 mm and D_{65} = 0.322 mm. Calculate Einstein's R' from the Manning-Strickler equation. Use the Einstein-Barbarossa graph to obtain a calculated value of the mean velocity, U, for this flow.

(8) What was the bed configuration for this flow: ripples and dunes, flat bed, or antidunes? Justify your answer.

(9) Now a uniformly distributed inflow of uniform sediment with D = 0.28 mm enters the flow across the water surface, and the bed starts to aggrade (i.e., rise). The rate of sediment "rain" onto the surface is 5.15 x 10^{-3} kg/m^2/s.

 (a) Derive an expression for the new distribution of sediment concentration.

 (b) How much is the suspended-sediment discharge increased by this "sediment rain"?

 (c) How fast does the bed aggrade (build up)? Let porosity = 0.3.

<PROBLEM K-17T>

The following data were measured at Section A-2 of the Rio Grande on 27 May 1958:

Q	=	10,000 cfs (water discharge);
B	=	270 ft (channel width);
d	=	4.80 ft (mean flow depth);
D_{50}	=	0.30 mm;
D_{35}	=	0.24 mm;
S	=	0.00080; and,
r_c	=	1,500 ft (centerline radius of a long bend)

(1) Calculate f/f' or f/f_f for this flow.

(2) What was the bed configuration for this flow? Give the basis for your answer.

(3) What was the wave length of the bed forms (if any) identified in Part (2) above?

(4) What were the *local* depths and velocities of flow at distances of 20 ft from the concave bank and from the convex bank?

(5) Using a *simple* formulation (*not*, e.g., the Einstein or Toffaleti procedure) of your choice, calculate the total sediment discharge of the river for the given conditions. Disregard curvature effects for this calculation. (The measured transport rate was 80,000 to 85,000 tons/day.)

(6) What is the largest bed material this flow could transport at each of the locations identified in Part (4), above?

(7) Is u/u_*'' for this flow consistent with the Einstein-Barbarossa bar-resistance graph? If not, what is the reason?

<PROBLEM K-18T>

One of the Pakistan LINK canals has the following operating characteristics (No. 10 of Brownlie's compilation, page 128B or page 176):

Q	=	72.6 m^3/s;
S	=	1.16×10^{-4};
C	=	142 ppm (total-load sediment concentration); and,
D_{50}	=	0.15 mm

Assume $f/f_0 = 4.5$

where $$f_0 = \frac{8}{\left\{6.25 + 2.5\ln\frac{d}{2.5D_{50}}\right\}^2} \qquad (1)$$

and utilize

$$\frac{U}{\sqrt{g(s-1)D_{50}}} = 8.68\left[\frac{q_s}{\sqrt{g(s-1)D_{50}^3}}\right]^{0.216} \qquad (2)$$

to design a canal (i.e., determine the width, depth, and velocity) for this flow.

[Hint]: Assume an initial value of channel width, guided by regime theory or other ideas. Then analyze the flow in a channel of that width, and adjust width, by iteration until a solution is arrived at.

<PROBLEM N-19T>

The following flow and bed-sediment data were collected by the IIHR river crew in Pool 20 of the Mississippi River (Keokuk, Iowa - Canton, Missouri) in May 1976. The specific gravity of the bed sediment can be assumed to be 2.65.

(1) Estimate the sediment discharge for each section using the Inglis-Lacey formula.

(2) Estimate the sediment discharge for each section using the Engelund-Hansen formula

(3) Estimate the sediment discharge for each section using the Einstein-Brown formula.

Section No.	Water Temperature T (°F)	Kinematic Viscosity ν (ft^2/s)	Flow Depth d (ft)	Mean Velocity V (ft/s)	Energy Slope S (ft/ft)	Median Particle Diameter D_s (ft)
1	54	1.40×10^{-5}	18.6	3.19	7.00×10^{-5}	2.26×10^{-3}
2	61	1.20×10^{-5}	13.7	2.75	5.69×10^{-5}	2.00×10^{-3}
3	62	1.18×10^{-5}	11.9	2.86	5.61×10^{-5}	2.13×10^{-3}
4	63	1.16×10^{-5}	12.0	2.58	6.63×10^{-5}	2.07×10^{-3}
5	65	1.13×10^{-5}	16.6	2.81	5.35×10^{-5}	1.25×10^{-3}
6	65	1.13×10^{-5}	14.1	2.57	5.61×10^{-5}	1.64×10^{-3}
7	67	1.10×10^{-5}	12.1	2.58	5.53×10^{-5}	1.77×10^{-3}
8	65	1.13×10^{-5}	10.2	1.98	3.84×10^{-5}	1.48×10^{-3}

<PROBLEM N-20T>

Flow and sediment discharges were measured by the IIHR river crew at Section 1-2 near Fox Island in Pool 20 of the Mississippi River on 7 May 1976. Bed-material samples from five different verticals along the cross section were sieve-analyzed as a composite sample. The result of the sieve analysis is shown below.

Fraction I.D. i	Sediment Size (mm)	Geometric Mean Size D_{si} (ft)	Size Fraction P_i (%)
1	32 16	7.42×10^{-2}	2.5
2	16 8	3.71×10^{-2}	7.2
3	8 4	1.86×10^{-2}	6.1
4	4 2	9.28×10^{-3}	6.5
5	2 1	4.64×10^{-3}	12.9
6	1.000 0.500	2.32×10^{-3}	33.1
7	0.500 0.250	1.16×10^{-3}	29.5
8	0.250 0.125	5.80×10^{-4}	2.0
9	0.125 0.062	2.89×10^{-4}	0.2

The measured hydraulic and sediment parameters of interest are as follows:

Water discharge: Q = 136, 500 cfs;
Mean flow velocity: V = 3.19 ft/s;
Water-surface slope: S = 7.000×10^{-5} ft/ft;
Channel width: W = 2,297 ft;
Hydraulic radius: R = 18.6 ft;
Water temperature: T = 54 °F ($\nu = 1.32 \times 10^{-5}$ ft²/s);
D_{50} = 2.26×10^{-3} ft;
D_{65} = 3.28×10^{-3} ft;
D_g = 3.87×10^{-3} ft; and
σ_g = 3.00

(1) Apply the Toffaleti formulas to estimate the suspended- and bed-load discharges for this section.

(2) The measured suspended- and bed-load discharges were Q_S = 77,100 tons/day and Q_B = 1,940 tons/day, respectively. By comparing your results with the measured discharges, discuss how accurate predictions the Toffaleti formulas can provide.

(3) Apply the Schoklitsch formula to estimate bed-load discharge for this section.

(4) Apply the Engelund-Fredsϕe formula to estimate the suspended- and bed-load discharges for this section.

Note: Fall velocity can be calculated using the Rubey formula for each sediment size:

$$w = F_1\sqrt{\left(\frac{\gamma_s}{\gamma}-1\right)gD_s} \qquad (1)$$

where
$$F_1 = \sqrt{\frac{2}{3}+\frac{36\nu^2}{\left\{gD_s^3(\frac{\gamma_s}{\gamma}-1)\right\}}}-\sqrt{\frac{36\nu^2}{\left\{gD_s^3(\frac{\gamma_s}{\gamma}-1)\right\}}} \qquad (2)$$

<PROBLEM N-21T>

The following field data were collected by the U.S.G.S. at Butte City in the Sacramento River on 16 December 1977:

Water temperature:	T	=	10 °C (ν = 1.41 x 10^{-5} ft²/s);
Water discharge:	Q	=	22,600 cfs;
Cross-section area:	A	=	6,090 ft²;
Mean flow velocity:	V	=	3.71 ft/s;
Water-surface width:	W	=	490 ft;
Mean flow depth:	d	=	12.4 ft;
Hydraulic radius:	R	=	12.3 ft;
Water-surface slope:	S	=	2.25 x 10^{-4} ft/ft;

Mean suspended-sediment concentration: C = 967 ppm;

D_{35} = 1.02×10^{-3} ft;
D_{50} = 1.54×10^{-3} ft;
D_{65} = 4.43×10^{-3} ft; and,
γ_s = 2.65

i	1	2	3	4	5	6	7	8	9	10
$D_{si} \times 10^4$ (ft)	742	371	186	92.8	46.4	23.2	11.6	5.8	2.89	1.46
P_i (%)	17.5	6.7	5.7	3.5	3.5	10.9	26.9	23.6	1.6	0.1

(1) Apply the Ackers and White formula to compute the total-load discharge and compare the result with the measured Q_s.

(2) Apply Einstein's bed-load function to calculate bed-load and suspended-load discharges. How does your Q_s compare with the measured Q_s?

<PROBLEM N-22T>

One of the lectures you have had in this course was related to "The Great Flood of '93." On the basis of the knowledge gained through that special presentation, answer the following questions:

(1) What were the major causes for the flood? List at least five contributing factors (please do not attempt to describe each in detail).

(2) What were the major long-term impacts of the flood in terms of sediment yields during the flood?

(3) What did the Interagency Committee on Floodplain Management recommend to President Bill Clinton with regard to flood plain management of the Upper Mississippi River and the Missouri River basins? Describe concisely in less than 100 words.

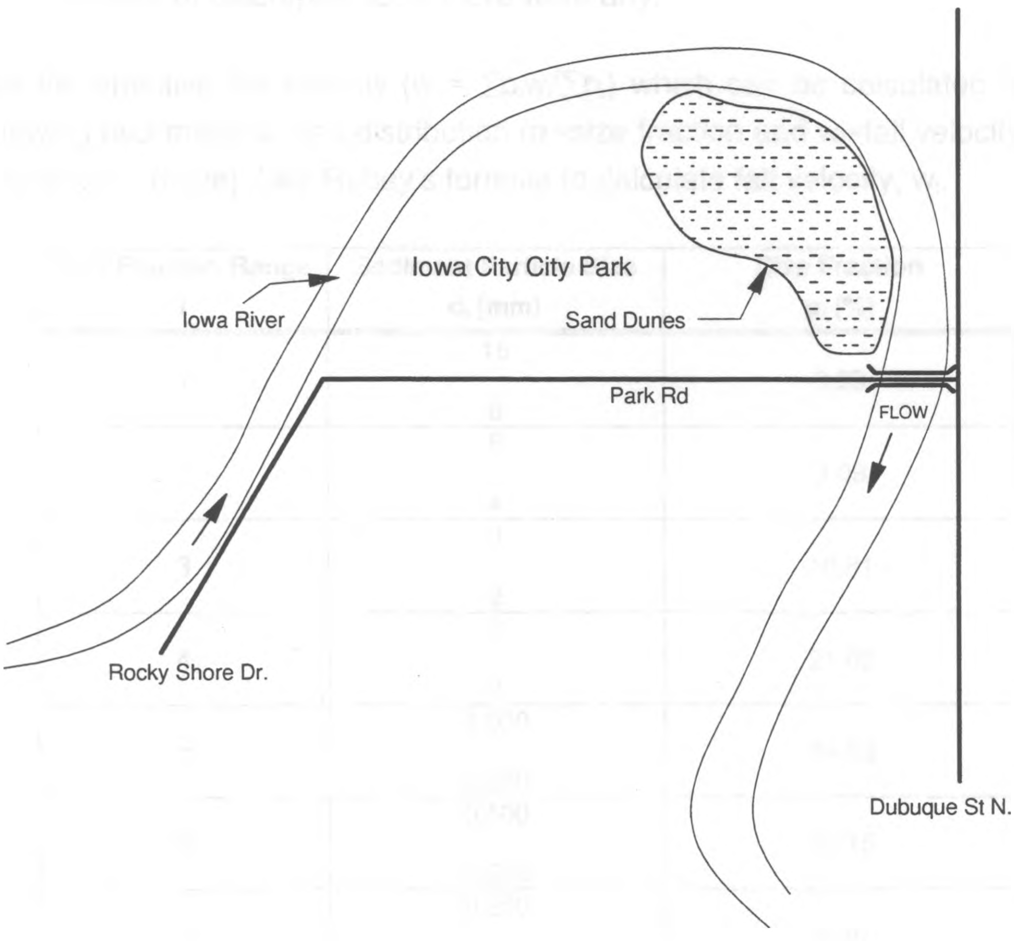
(4) Look at the Iowa River flow through the window, and estimate the suspended-sediment concentration and the water discharge; hence, calculate the suspended-sediment discharge (in tons/day). Doesn't regular exercise in each sediment transport class to predict these parameters help you?

<PROBLEM N-23T>

While inspecting the flood damage in City Park in Iowa City immediately after the Flood of '93, the park was found to be covered by large-scale sand dunes composed of rather uniform ***medium-size sand***. Water marks on trees showed that flow depth over the dunes was about 5 ft. As shown in the following sketch, the park is located in the inside bend of the Iowa River.

(1) Explain how such sand dunes were formed in the floodplain in the park.

(2) A beautiful scour hole, similar to scour holes around bridge piers, was detected around a 4-ft diameter oak tree which was buried partially by a 3-ft high dune.. The maximum scour depth was measured to be about 18 in. Estimate the mean flow velocity which produced this scour hole.



<PROBLEM N-24T>

The following data were collected from the Wapsipinicon River by the U.S.G.S. on 21 March 1979 at DeWitt, Iowa:

Energy slope:	$S = 3.22\text{x}10^{-3}$;	Flow depth:	$D = 7.2$ ft;
Mean velocity:	$V = 5.47$ ft/s;	Water temp:	$T = 8.5°C$;
Channel width:	$W = 310$ ft:	Median diameter:	$d_{50} = 0.78$ mm;
$d_{16} = 0.36$ mm;	$d_{84} = 1.9$ mm;		

Bed-load discharge: $Q_B = 1{,}370$ tons/day; and,
Suspended-load discharge: $Q_s = 17{,}700$ tons/day

(1) Using Engelund and Fredsϕe's formula, estimate both the bed-load and suspended-load discharges.

(2) Compare your results with the measured values and discuss primary causes of discrepancies if there were any.

Use the effective fall velocity ($w = \Sigma p_i w_i / \Sigma p_i$) which can be calculated from the following bed-material size distribution (p_i=size fraction and w_i=fall velocity for i-th size range): [Note] Use Rubey's formula to calculate fall velocity, w_i.

Size Fraction Range i	Sediment Particle Size d_i (mm)	Size Fraction p_i (%)
1	16 – 8	0.39
2	8 – 4	3.09
3	4 – 2	10.81
4	2 – 1	21.63
5	1.000 – 0.500	34.53
6	0.500 – 0.250	23.15
7	0.250 – 0.125	6.40

<PROBLEM N-25T>

The following field data were collected from the Missouri River near Council Bluffs, Iowa by the IIHR river crew on 30 November 1982 during a diagnostic study of Iowa Generation's Council Bluffs Station Unit 3 water-intake shoaling problems:

Distance from left bank z (ft)	Flow depth D (ft)	Mean flow velocity V (ft/s)	Depth-averaged suspended-sediment concentration C (ppm)	D_{35} (mm)	D_{50} (mm)	σ_g
49	10.9	4.74	261	0.380	0.480	2.04
105	19.4	4.56	370	0.580	0.650	1.59
230	24.5	6.50	760	---	---	---
295	14.4	5.62	987	0.202	0.218	1.31
361	15.3	4.00	546	---	---	---
443	12.2	5.40	847	0.195	0.216	1.31
525	12.6	5,40	738	---	---	---
623	9.9	1.76	397	0.217	0.242	1.42
689	1,8	0.83	---	---	---	---

Note: Water temperature = 4 °C; energy slope = 1 ft/mile; and channel width = 719 ft.

(1) Is the left bank convex?

(2) Calculate total water discharge, Q, and total suspended-load discharge, Q_S.

(3) What are the mean flow depth, d_m, mean flow velocity, V_m, and hydraulic radius, R for this section?

(4) Estimate the total-load discharge, Q_T, using the Ackers and White formula.

(5) Estimate the hydraulic radius using Brownlie's method and compare the result with the field-data based value obtained above.

<PROBLEM N-26T>

The one-dimensional shallow-water equations of motion and mass conservation are given by

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+g\frac{\partial}{\partial x}(h+e)+c_b\frac{u|u|}{h}=0 \tag{1}$$

$$\frac{\partial h}{\partial t}+\frac{\partial}{\partial x}(uh)=0 \tag{2}$$

where u is the mean longitudinal flow velocity, h is the flow depth, c_b is the friction coefficient, and e is the local bed elevation. The equation of the mass conservation of the bed is given by

$$\frac{\partial q_b}{\partial x}+\frac{\partial e}{\partial t}=0$$

or

$$q_1(u,h)\frac{\partial u}{\partial x}+q_2(u,h)\frac{\partial h}{\partial x}+\frac{\partial e}{\partial t}=0 \tag{3}$$

where $q_1=\frac{\partial q_b}{\partial u}$ and $q_2=\frac{\partial q_b}{\partial h}$, assuming that $q_b = q_b(u,h)$.

Using nondimensional constants, u'=u/U, h'=h/H, e'=e/H, x'=x/L, t'=t/T, and q_b'=q_b/Q_b, obtain a set of dimensionless equations which are given as follows:

$$\boldsymbol{A}\frac{\partial \boldsymbol{v}'}{\partial x'}+\boldsymbol{B}\frac{\partial \boldsymbol{v}'}{\partial t'}+\boldsymbol{C}=0 \tag{4}$$

where

$$\boldsymbol{A}=\begin{bmatrix} F^2u' & 1 & 1 \\ h' & u' & 0 \\ q_1'M & q_2'M & 0 \end{bmatrix} \quad \boldsymbol{B}=\begin{bmatrix} F^2T & 0 & 0 \\ 0 & T & 0 \\ 0 & 0 & T \end{bmatrix} \tag{5}$$

$$\boldsymbol{v}'=\begin{bmatrix} u' \\ h' \\ e' \end{bmatrix} \qquad \boldsymbol{C}=\begin{bmatrix} J\dfrac{u'|u'|}{h'(1+n)} \\ 0 \\ 0 \end{bmatrix}$$

where

$$q_1' = \frac{1}{Q_b}\frac{\partial q_b}{\partial u'};\quad q_2' = \frac{1}{Q_b}\frac{\partial q_b}{\partial h'};\quad \boldsymbol{F}^2 = \frac{U^2}{gH};\quad \boldsymbol{T} = \frac{L}{UT};$$

$$\boldsymbol{J} = \frac{c_b L U^2}{qH^2};\quad \boldsymbol{M} = \frac{Q_b}{UH};\quad c_b = \left(\frac{H}{h}\right)^n C_b; \tag{6}$$

$$\text{and}\quad Q_b = \chi\left[C_b \rho U^2 - \tau_0\right]^r = \chi\left[\rho g H S_0 - \tau_0\right]^r$$

<PROBLEM N-27T>

Equation (4) in Problem N-26T has characteristic curves which are the solutions of the ordinary differential equations,

$$\left(\frac{\partial x'}{\partial t'}\right)_i = \frac{T}{L}\left(\frac{\partial x}{\partial t}\right)_i = s_i \quad (i = 1, 2, 3) \tag{1}$$

(1) Show that the secular equation of $|\boldsymbol{A} - s\boldsymbol{B}|$ is given by

$$\boldsymbol{F}^2(\boldsymbol{T}^3 s^3 - 2u'\boldsymbol{T}^2 s^2) - \boldsymbol{T}s(q_1'\boldsymbol{M} + h' - u'^2\boldsymbol{F}^2) + (q_1'u' - q_2'h')\boldsymbol{M} = 0 \tag{2}$$

(2) Show that the characteristics (1) are given by the following equations if $q_1' = q_2' = 0$:

$$\left(\frac{\partial x}{\partial t}\right)_{1,2} = u \pm (gh)^{1/2} \quad \text{(celerity of surface wave propagation over a rigid bed)} \tag{3}$$

$$\left(\frac{\partial x}{\partial t}\right)_3 = 0 \tag{4}$$

<PROBLEM N-28T>

Now let us expand Problem N-27T to the case of nonzero sediment-transport rate. With the assumption that $\boldsymbol{M} << 1$, let us expand s_i in powers of $\boldsymbol{M}$:

$$s_i = s_i^{(0)} + \boldsymbol{M} s_i^{(1)} + \boldsymbol{M}^2 s_i^{(2)} + \cdots \tag{1}$$

where subscript, i, is used to number the roots, which gives the order of the approximation. By substituting this series into equation (2) of Problem N-27T, obtain the following characteristics which give celerities of disturbance in an erodible bed channel.

$$\left(\frac{dx}{dt}\right)_1 = v_1 = u + (gh)^{1/2} + \frac{q_1'\boldsymbol{F} + q_2'\boldsymbol{F}^2}{2\boldsymbol{F}^2 + 2\boldsymbol{F}^3}\boldsymbol{M}u + O(\boldsymbol{M}^2) \tag{2}$$

$$\left(\frac{dx}{dt}\right)_2 = v_2 = u - (gh)^{1/2} - \frac{q_1'\boldsymbol{F} - q_2'\boldsymbol{F}^2}{2\boldsymbol{F}^2 - 2\boldsymbol{F}^3}\boldsymbol{M}u + O(\boldsymbol{M}^2) \tag{3}$$

$$\left(\frac{dx}{dt}\right)_3 = v_b = \frac{q_1' - q_2'}{1 - \boldsymbol{F}^2}\boldsymbol{M}u + O(\boldsymbol{M}^2) \tag{4}$$

Note that u'=h'=1 and Q_b=q_b for the undisturbed, steady and uniform flow.

<PROBLEM N-29T>

Let us consider a steady flow u' = h' = e' = constant which is perturbed in such a way that the new flow variables are u'+δu', h'+δh', and e'+δe'. If the original equations of the undisturbed flow are subtracted from the equations of the perturbed flow in equation (4) of Problem N-26T, a system of partial differential equations in terms of disturbances δu', δh', and δe' is obtained. Show that it is written in the matrix form:

$$K\delta V' = 0 \quad (1)$$

where

$$K = \begin{bmatrix} F^2\left[T\dfrac{\partial}{\partial t'} + \dfrac{\partial}{\partial x'}\right] + 2J & \dfrac{\partial}{\partial x'} - (1+n)J & \dfrac{\partial}{\partial x'} \\ \dfrac{\partial}{\partial x'} & T\dfrac{\partial}{\partial t'} + \dfrac{\partial}{\partial x'} & 0 \\ 2rM^*\dfrac{\partial}{\partial x'} & -nrM^*\dfrac{\partial}{\partial x'} & T\dfrac{\partial}{\partial t'} \end{bmatrix} \quad (2)$$

where

$$\delta v' = [\delta u', \delta h', \delta e'] \text{ and } M^* = M/(1-H) \text{ in which } H = \tau_0/(c_b \rho U^2) \quad (3)$$

[Hints]: 1. Neglect terms $O(\delta u'^2)$, ..., and higher order terms.

2. After subtraction, set u'=h'=1 and Q_b=q_b for the steady uniform flow.

<PROBLEM N-30T>

Since equation (1) in Problem N-29T is a linear system, it can be reduced to a single equation by setting $|K|$=0,

$$\begin{aligned}&(2+n)rM^*\frac{\partial^3\phi}{\partial x'^3} - F^2T^3\frac{\partial^3\phi}{\partial t'^3} + (1+2rM^* - F^2)T\frac{\partial^3\phi}{\partial t'\partial x'^2} - 2F^2T^2\frac{\partial^3\phi}{\partial t'^2\partial x'} \\ &-2JT^2\frac{\partial^2\phi}{\partial t'^2} - (3+n)JT\frac{\partial^2\phi}{\partial t'\partial x'} = 0\end{aligned} \quad (1)$$

where ϕ stands for $\delta u'$, $\delta h'$, and $\delta e'$.

(1) Drive equation (1) above.

(2) Let us consider a neutral disturbance of the form

$$\phi = \phi^* \exp[ik(x-vt)] \quad (2)$$

where $k = 2\pi/l$ (wave number), l = wave length, v = wave celerity, and ϕ^* = amplitude of ϕ.

Substitute equation (2) into equation (1) and obtain the following relationships:

$$v = \frac{(3+n)u}{2} \quad \text{(from real part)} \tag{3}$$

$$\boldsymbol{F}^2 = \frac{4}{(1+n)^2} + \frac{8}{(3+n)(1+n)^2} r\boldsymbol{M}^* \quad \text{(from imaginary part)} \tag{4}$$

[Hint]: Take k^{-1} as the characteristic length so that $\boldsymbol{T}$=1/(kuT) and $\boldsymbol{J}$=S_0/(kh).

<PROBLEM N-31T>

Assume that the continuity equation for the one-dimensional sediment-laden flow is given by

$$\frac{1}{B}\frac{\partial Q}{\partial x} + \frac{\partial y}{\partial t} = \frac{q}{B} \qquad \text{(Q and y = unknown)} \tag{1}$$

where

Q = flow rate of water-sediment mixture;
B = water-surface width = function of y = B(y);
y = flow depth; and,
q = lateral water inflow

(3) Using Preissman's implicit finite difference scheme, discretize equation (1), and rearrange the equation into the following form:

$$A\Delta y_j + B\Delta Q_j + C\Delta y_{j+1} + D\Delta Q_{j+1} + H = 0 \tag{2}$$

In determining A, B, ..., H, you can neglect second-order terms.

(4) Assuming that the upstream and downstream boundary conditions are respectively given by

$$Q_1 = f(y_1) \qquad \text{(upstream boundary condition)} \tag{3}$$

$$Q_N = g(y_N) \qquad \text{(downstream boundary condition)} \tag{4}$$

Determine E_1 and E_2 in terms of $f(y_1)$; and Δy_N in terms of $g(y_N)$, E_N, and F_N.

[Hint]: Assume that $\Delta Q_j = E_j \Delta y_j + F_j$

<PROBLEM N-32T>

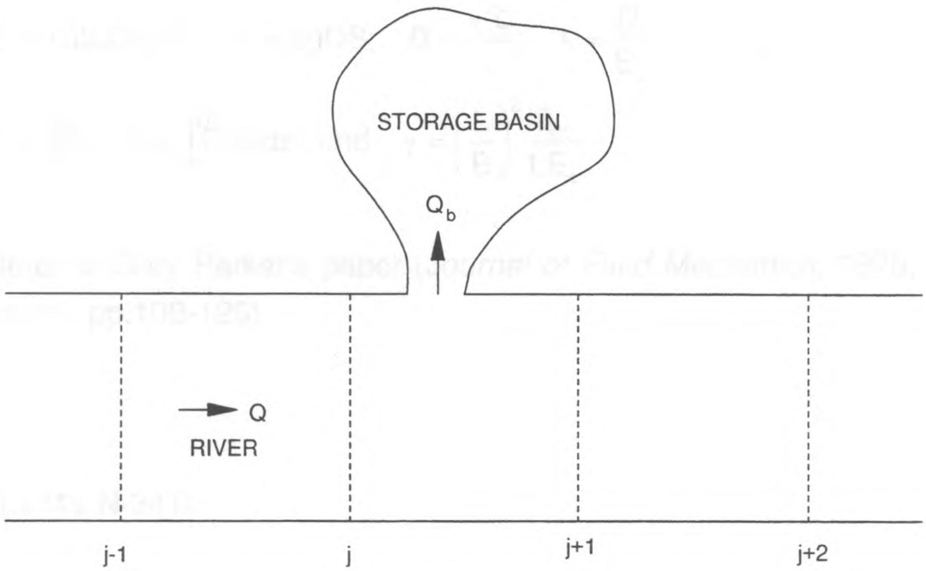
Assume that the following linear relationship holds between Δy_j and ΔQ_j for any $x = j\Delta x$ for a particular time step:

$$\Delta Q_j = E_j \Delta y_j + F_j \tag{1}$$

where E_j and F_j = coefficients; y = water-surface elevation; and Q = water discharge.

Now consider the river reach with a storage basin, as sketched below, and obtain E_{j+1} and F_{j+1} in terms of E_j, F_j, Δt, y_j^n, y_{j+1}^n, Q_j^n, Q_{j+1}^n, and A_b, where $A_b = A_b(y_b)$ = water-surface area of the basin.

[Hint]: Assume that the water-surface elevation at the storage basin, y_b, is equal to that at the river section.



<PROBLEM N-33T>

The transverse variation of the vertically-integrated lateral suspended-sediment flux is balanced by a net erosion rate, i.e.,

$$\frac{dF_L}{dy} = E - \boldsymbol{D} \qquad (1)$$

Starting from this equation, obtain the governing equation for $z = (\zeta/\zeta_c)$,

$$\frac{\gamma}{K}\frac{d^2z}{d\eta^2} = \lambda Z - G^{3/4} \qquad (2)$$

where

$$s = \frac{D}{D_c};\quad G = s^4;\quad \eta = \frac{y}{B};\quad F_L = -\varepsilon_y \frac{d\zeta}{dy};\quad \overline{F}_L = \varepsilon_y \frac{\zeta_c}{L};$$

$$E = 0.0233 v_s \tau_*^3;\quad \tau_* = \rho g D S;\quad \boldsymbol{D} = \frac{v_s^2 \zeta}{\varepsilon};\quad \lambda = \frac{\boldsymbol{D}_c}{E_c};\qquad (3)$$

$$K = \frac{\overline{q}_{BL}}{\overline{F}_L};\quad \zeta = \int_a^D C(z)dz;\text{ and }\quad \gamma = \left(\frac{L}{B}\right)^2 \frac{\overline{q}_{BL}}{LE_c}$$

Note: Refer to Gary Parker's paper (*Journal of Fluid Mechanics,* 1978, Vol. 89, Part 1, pp.109-125)

<PROBLEMS N-34T>

The following data were obtained at Section C-6 of the Niobrara River on 3 August 1951 (Colby and Hembree, 1955):

Cross-section profile

(y: distance from the right bank and D: flow depth)

y (ft)	0	6	14	16	22	24	30	32	40	42	48	50	54
D (ft)	0	0.9	1.2	1.3	1.2	1.4	1.4	1.4	1.6	1.8	2.0	1.9	1.6
y (ft)	60	64	70	76	82	89	95	110	124	132	134		
D (ft)	1.7	1.7	1.3	1.6	1.0	0.4	0.3	0.3	0.6	0.8	0		

Bed-material size distribution

Size (mm)	0.125	0.250	0.500	1.000	2.000	4.000
% finer	2	37	71	92	96	98

River width = 134 ft; mean flow depth = 1.52 ft; water temperature = 23 °C; flow area = 138 ft^2; mean velocity = 2.53 ft/s; water discharge = 349 ft^3/s; and water-surface slope = 1.402x10^{-3}.

(1) Plot the cross section and the cumulative sediment particle size distribution, and determine approximately the center depth, D_c, L (bank region), and D_{50}.

(2) Assuming that the channel is in equilibrium (i.e., $\boldsymbol{D}_c = E_c$), calculate $\overline{F}_L$ and $\overline{q}_{BL}$. Is your K value ($\overline{F}_L/\overline{q}_{BL}$) close to 24/7? You can assume that D_s=D_{50}.

(3) Evaluate γ for this case and see how small this parameter is – justification of the asymptotic expansion in terms of γ.

Note: Refer to Gary Parker's paper (*Journal of Fluid Mechanics,* 1978, Vol. 89, Part 1, pp.109-125)

<PROBLEM N-35T>

In one-dimensional shallow-water analysis, the governing equations for a movable-bed channel with varying width may be given in the following form:

$$\frac{1}{g}\frac{\partial v}{\partial t}+\frac{v}{g}\frac{\partial v}{\partial x}+\frac{\partial D}{\partial x}+\frac{\partial z}{\partial x}+\frac{u_*^2}{gR}=0 \tag{1}$$

$$\frac{\partial A}{\partial t}+\frac{\partial Q}{\partial x}=0 \quad \text{or} \quad \frac{\partial(\ln D)}{\partial t}+\frac{\partial(\ln W)}{\partial t}=-\frac{1}{A}\frac{\partial Q}{\partial x}; \quad (A=WD) \tag{2}$$

$$\frac{\partial z}{\partial t}-\frac{z_0-z}{W}\frac{\partial W}{\partial t}+\frac{\partial q_s}{\partial x}=0 \tag{3}$$

$$q_s=q_s(v,D) \quad \text{or}$$
$$\frac{\partial q_s}{\partial x}=q_1\frac{\partial v}{\partial x}+q_2\frac{\partial D}{\partial x}; \quad \left(q_1=\frac{\partial q_s}{\partial v} \text{ and } q_2=\frac{\partial q_s}{\partial D}\right) \tag{4}$$

$$\frac{\partial W}{\partial x}=K\frac{\partial D}{\partial x} \quad \text{(assumed)} \tag{5}$$

where

v = mean flow velocity; D = flow depth; z = bed elevation; u_* = shear velocity; R = hydraulic radius (≅ D); A = cross-section area; Q = water discharge; W = channel width; z_0 = bank elevation; q_s = sediment discharge per unit width; and K = coefficient.

Introduce nondimensional parameters, v'=v/U; d'=D/H; z'=z/H; W'=W/H; x'=x/L; t'=t/T; and q_s'=q_s/Q_s), and obtain a set of dimensionless equations in the following form:

$$\boldsymbol{A}\frac{\partial \boldsymbol{V}'}{\partial x'}+\boldsymbol{B}\frac{\partial \boldsymbol{V}'}{\partial t'}+\boldsymbol{C}=0 \tag{6}$$

$$\boldsymbol{V}'=\begin{bmatrix} v' \\ D' \\ z' \\ W' \end{bmatrix} \tag{7}$$

You can use the following parameters when determining matrices ***A***, ***B***, and ***C***:

$$F^2=\frac{U^2}{gH};\quad T=\frac{L}{UT};\quad P=\frac{C_bLU^2}{gH^2};\quad M=\frac{Q_s}{UH};$$
$$q_1'=\frac{1}{q_s}\frac{\partial q_s}{\partial v'};\quad q_2'=\frac{1}{Q_s}\frac{\partial q_s}{\partial D'};\quad c_b=\left(\frac{H}{D}\right)^n C_b;\quad \text{and}\quad u_*^2=c_b v|v| \tag{8}$$

<PROBLEM N-36T>

A one-dimensional analysis of the diffusion equation,

$$\frac{\partial z}{\partial t}=K_0\frac{\partial^2 z}{\partial x^2} \tag{1}$$

gives the following solution:

$$\zeta = \xi - 2(1-\gamma)\sum_{n=1}^{\infty}\left[\left(\frac{1}{\beta_n}\right)^3 \sin(\beta_n \xi)\exp(-\beta_n{}^2\eta)\right] \tag{2}$$

where

$$\gamma = \frac{S_0}{S_\ell}; \quad S_0 = a_1; \quad S_\ell = a_1 + 2a_2\ell; \quad \varsigma = \frac{z}{\ell S_\ell}; \quad \xi = \frac{x}{\ell};$$
$$\eta = \frac{K_0 t}{\ell^2}; \quad \text{and} \quad \beta_n = \frac{(2n-1)}{n}\pi \tag{3}$$

(1) Calculate ς for different values of η (10^{-3} ~ 1) four different sets of ξ (i.e., 0.25, 0.50, 0.75, and 1.00), and plot the results on an η-ζ plane with a parameter ξ. Assume that γ is equal to 0.22 ($S_0 = 2.1\text{x}10^{-3}$; and S_ℓ = $9.5\text{x}10^{-3}$).

(2) Estimate values of η for the following set of field data and obtain a relationship between η and t.

Date Surveyed	Distance, x (m)	Bed Elevation, z (m)
March 1961 (t = 0)		
March 1962	1,500	5.4
March 1963	1,500	6.0
March 1964	6,000	36.6
October 1964	3,000	14.3
September 1966	4,500	26.3

Note: ℓ = 6 km and S_ℓ = $9.5\text{x}10^{-3}$

(3) Is it possible to determine a bed-load discharge formula which is similar to the Kalinske-Brown type formula if enough field data on x, z, and t were given? If your answer is yes, describe what other parameters have to be measured in the field.

<PROBLEM N-37T>

The following data were obtained by the IIHR river crew at Section 1-2 in Pool 20 of the Mississippi River on 7 May 1976:

$V = 3.19$ ft/s; $u_* = 0.205$ ft/s; $T = 54$ °F; $\gamma_s/\gamma = 2.65$; $d_s = 2.26\text{x}10^{-3}$ ft; $D = 18.6$ ft

(1) Using Engelund-Hansen's formula, estimate the total-load discharge per unit width, q_b.

(2) For navigation purposes, a 5-ft deep, 250-ft wide (base) trapezoidal channel was provided on the bottom of the Mississippi River by a dredging operation. The side slope of the channel was 1V:2H. Using Fredsφe's formula (*ASCE Proc.*, Vol. 104, No. HY2, 1978), estimate the bed profiles for t = 1 hr, 10 hrs, 1 day, and 10 days, and plot the results in one graph for comparison. Assume that the dynamic friction angle is 27°, and porosity of the bed material is 0.45.

(3) Suppose that the cross section of Section 1-2 is in a stable condition under the given hydraulic conditions. Estimate the flow depth at the center region using Parker's result (*Journal of Fluid Mechanics,* 1978, Vol. 89, Part 1, pp.109-125).

VII. SEDIMENT DISCHARGE IN PIPES

<PROBLEM V-1SP>

The literature on transportation of sediments in pipes defines the following regimes of flow:

(a) Heterogeneous flow is one in which all the sediments in the system are in suspension but the concentration is not uniform.

(b) Homogeneous flow is one in which all the sediment in the system is in suspension and the concentration is uniform.

Discuss the following:

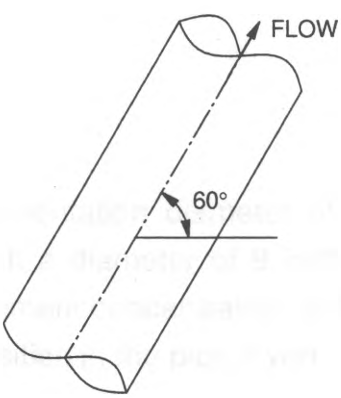
(1) Is it possible to have a sediment-laden flow with all the sediment in suspension, i.e., with no bed load?

(2) Under what conditions can we have a uniform concentration of suspended sediment?

(3) Giver your interpretation of the terms "heterogeneous flow" and "homogeneous flow"

<PROBLEM K-2SP>

A sediment-laden flow is transported up an inclined pipe, as shown below.



The inside diameter of the pipe is 1.0 ft; the transported sand has a diameter of 0.20 mm; the water temperature is 20 °C; and m = 0.1. The mean velocity of the mixture is 10 ft/s, and the Darcy-Weisbach friction factor is 0.025.

(1) Find the maximum sediment transport rate that can be achieved without bed deposition (i.e., keeping $\theta_b = 0$). For deposited material, C = 1,850 gm/l (grams of sediment per liter of mixture). Do not neglect the settlement of the material along the pipe.

(2) Calculate the water discharge.

Note: $$I_1(x) \cong \frac{e^x}{\sqrt{2\pi x}}\left[1 - \frac{3}{8x} - \frac{15}{128x^2}\right]$$

<PROBLEM K-3SP>

Water with a temperature of 80 °F flows through a horizontal circular 8-inch diameter pipe at a rate of 3.50 cfs (water discharge).

(1) Estimate the maximum discharge of sand (Dg = 0.15 mm) that can be conveyed without significant bed deposition (i.e., with $\theta_b = 0$).

(2) Using Durand's curve, estimate the energy gradient of the flow.

<PROBLEM K-4SP>

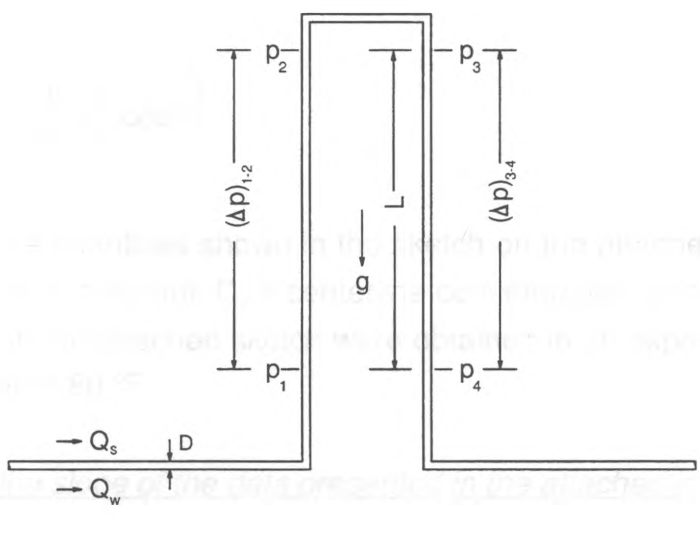
Uniform sand with a sedimentation diameter of 0.08 mm is transported in a horizontal circular pipe with a diameter of 9 inches. The mean velocity of the flow is 12 ft/s, and the sediment concentration at the invert of the pipe is 0.10 by volume. There is no deposition in the pipe invert. The following values apply:

m	=	0.10;
κ	=	0.25;
f	=	0.035; and,
T	=	20 °C

Find the sediment discharge in the pipe.

<PROBLEM K-5SP>

The attached figure shows a scheme that has been proposed for measurement of sediment concentration in slurry lines. The mixture is pumped through a pipe loop with two vertical legs, and the pressure drops, $(\Delta p)_{1-2}$ in the ascending leg and $(\Delta p)_{3-4}$ in the descending leg, are measured and compared. The sediment concentration is determined from the difference between those two pressure drops.



(1) For the situation shown in the sketch, which will be greater, $(\Delta p)_{1-2}$ or $(\Delta p)_{3-4}$? Justify your answer.

(2) Find the difference between $(\Delta p)_{1\text{-}2}$ and $(\Delta p)_{3\text{-}4}$ for the following conditions:

L	=	30 ft;
D	=	0.5 ft (inside pipe diameter);
f	=	0.04;
Q_s/Q_w	=	0.15;
V	=	15 ft/s (mean velocity of mixture);
Fluid	=	water; and,
Sediment	=	sand

Assume the flow is fully developed at all sections between the piezometers.

<PROBLEM K-6SP>

An analysis yields the following relation for the distribution of suspended sediment in a circular horizontal pipe, flowing full, without deposition of material in the pipe:

$$\frac{C}{C_0} = \exp\left[\frac{w}{mu_*}\frac{r}{r_0}\cos\theta\right] \qquad (1)$$

where, in addition to the quantities shown in the sketch on the attached figure, w = particle fall velocity; m = constant; C_0 = centerline concentration; and u_* = 0.363 ft/s. The data shown in the attached sketch were obtained in an experiment with 0.16 mm sand in water at 80 °F.

(1) Find m, <u>using the slope of the data presented in the attached graph.</u>

(3) Find Q_s and Q_w. Assume κ = 0.35 and B = 5.5C_0 = 6,020 ppm; f = 0.0159; r_0 = 2 in.; and θ_b = 0 for the data shown in the graph.

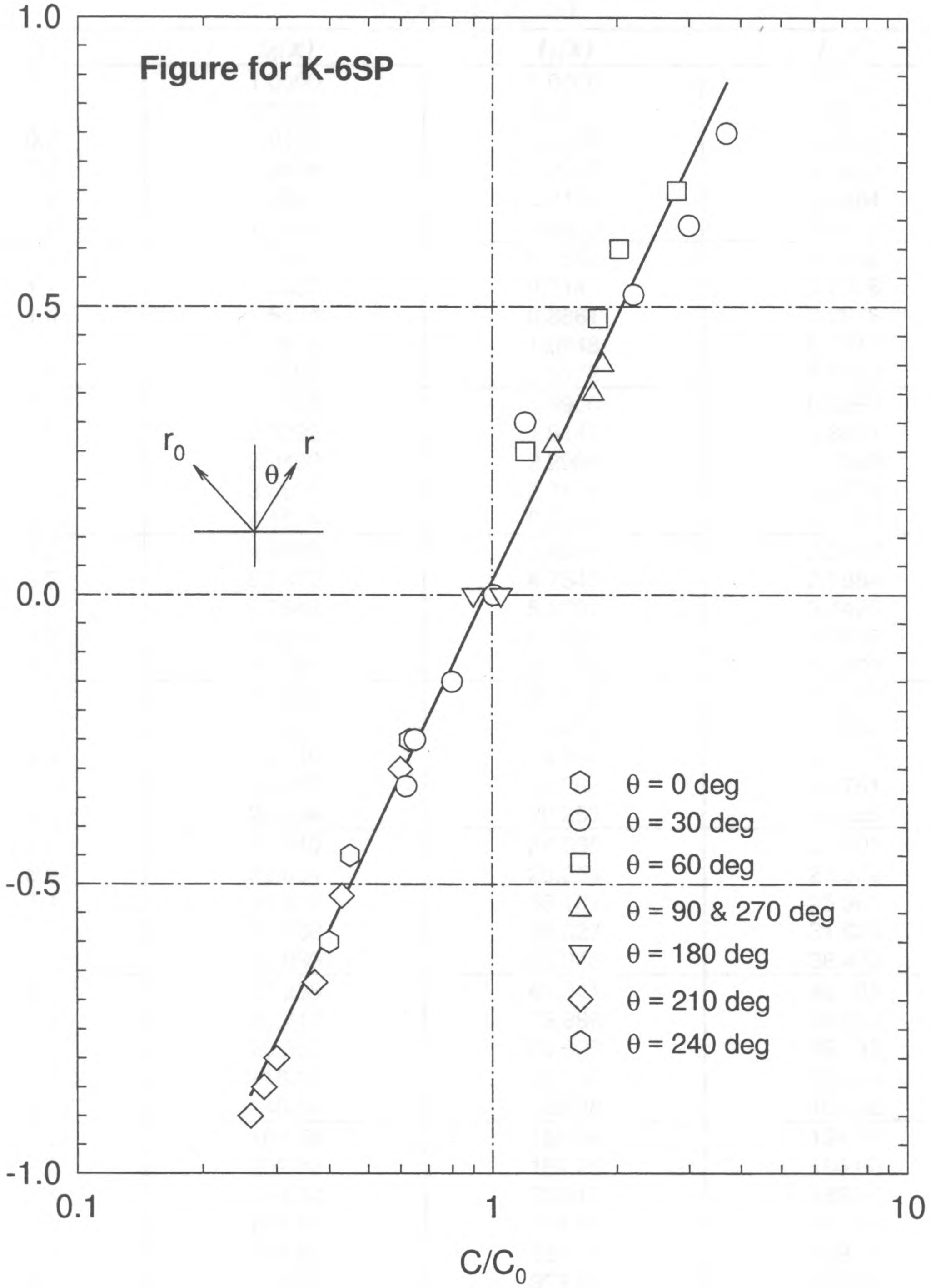


Figure for K-6SP
1.0
0.5
0.0
-0.5
-1.0
0.1
1
10
C/C0
r0
r
θ
θ = 0 deg
θ = 30 deg
θ = 60 deg
θ = 90 & 270 deg
θ = 180 deg
θ = 210 deg
θ = 240 deg

HYPERBOLIC BESSEL FUNCTIONS

$$I_m(x) = i^{-m} J_m(ix)$$

x	*I₀(x)*	*I₁(x)*	*I₂(x)*
0.0	1.0000	0.0000	0.0000
0.1	1.0025	0.0501	0.0012
0.2	1.0100	0.1005	0.0050
0.4	1.0404	0.2040	0.0203
0.6	1.0921	0.3137	0.0464
0.8	1.1665	0.4329	0.0843
1.0	1.2661	0.5652	0.1358
1.2	1.3937	0.7147	0.2026
1.4	1.5534	0.8861	0.2876
1.6	1.7500	1.0848	0.3940
1.8	1.9895	1.3172	0.5260
2.0	2.2796	1.5906	0.6890
2.2	2.6292	1.9141	0.8891
2.4	3.0492	2.2981	1.1342
2.6	3.5532	2.7554	1.4338
2.8	4.1574	3.3011	1.7994
3.0	4.8808	3.9534	2.2452
3.2	5.7472	4.7343	2.7884
3.4	6.7848	5.6701	3.4495
3.6	8.0278	6.7926	4.2538
3.8	9.5169	8.1405	5.2323
4.0	11.302	9.7594	6.4224
4.2	13.443	11.705	7.8683
4.4	16.010	14.046	9.6259
4.6	19.093	16.863	11.761
4.8	22.794	20.253	14.355
5.0	27.240	24.335	17.505
5.2	32.584	29.254	21.332
5.4	39.010	35.181	25.980
5.6	46.738	42.327	31.621
5.8	56.039	50.945	38.472
6.0	67.235	61.341	46.788
6.2	80.717	73.886	56.882
6.4	96.963	89.025	69.143
6.6	116.54	107.31	84.021
6.8	140.14	129.38	102.08
7.0	168.59	156.04	124.01
7.2	202.92	188.25	150.63
7.4	244.34	227.17	182.94
7.6	294.33	274.22	222.17
7.8	354.68	331.10	269.79
8.0	427.57	399.87	327.60

<PROBLEM K-7SP>

The Consolidation Company's coal slurry pipeline in Ohio has the following operating characteristics:

Material:	coal;
Length:	108 miles;
Capacity:	1.30×10^6 ton/year;
Particle size:	0.14 mm;
σ_g	1.2 (assumed);
Specific gravity:	1.40; and,
Operating factor*:	90% (assumed)
(* % of time pipeline is operating)	

Using the curves and equations presented in your reading assignment in ASCE Sedimentation Manual No. 54, design a slurry line to meet the above requirements. The design parameters should include, but not be limited to,

Pipeline inside diameter;
Water discharge;
Solid concentration;
Velocity; and,
Pumping power requirements

There must be no sediment deposition in the pipe invert. The flow is to be heterogeneous, and the design must be efficient. Clearly describe the procedure you use.

<PROBLEM K-8SP>

Slurry-transport pipelines with an equilateral triangle cross-section, as shown, have been proposed for practical use. A very reasonable assumption is that the sediment concentration is given by

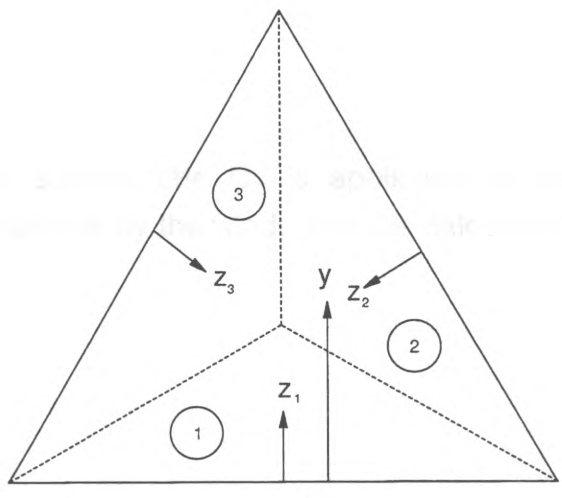
$$C = C_b \exp\left[-\frac{wy}{mu.b}\right] \qquad (1)$$

(1) Propose a reasonable velocity distribution (i.e., give the equations of u(z) for the three subsections shown in the sketch, using the "law of the wall", which states that the velocity at any point is determined by the distance z to the nearest wall, as well as by u_*, etc. Note that the same velocity distribution may be applied to all three subsections.

(2) Set up the integrals which give the sediment discharge. Do *NOT* carry out the integration(s)!!

(3) What might be the advantage of using a pipe of this form?

(4) What are the practical disadvantage of this pipe shape?



In subsection 1, z_1 = y

In subsection 2, z_2 = f(y) ----- to be determined by trigonometry

In subsection 3, z_3 = g(y) ----- to be determined by trigonometry

VIII. WIND-DRIVEN SEDIMENT TRANSPORT

<PROBLEM K-1A>

(1) Calculate the trajectory of a wind-transported sediment particle 0.25 mm in diameter assuming that it is projected vertically upward from the bed with an initial vertical velocity of $w_i = 0.8u_*$. Assume further that the velocity profile is given by

$$\frac{u}{U_0} = \left(\frac{y}{y_0}\right)^{\frac{1}{n}} \tag{1}$$

where n = 10 and the reference velocity is $U_0 = 10$ m/s at $y_0 = 1$ m.

(2) Generalize Nunner's law

$$\sqrt{f} = \frac{1}{n} \tag{2}$$

to calculate the shear velocity,

$$f = 8\left(\frac{yu}{U_0}\right)^2 \tag{3}$$

Assume that a steady-state C_D is applicable in calculating the force exerted on the particle by the wind. Plot the calculated trajectory to scale.

<PROBLEM K-2A>

Calculate the total momentum exchanged between the air and the particle in Problem K-1A.

<PROBLEM K-3A>

Wind blows over a desert sand bed. The sand is quartz with a uniform diameter of 0.25 mm.

(1) Calculate the bed shear velocity at which particle transport begins.

(2) Prepare a curve of sediment-transport rate per unit width vs. wind velocity measured 1 m above the bed, for velocities up to 30 m/s

IX. RIVER MEANDERING

<PROBLEM K-1M>

A curve in the river described in Problem K-1T has a centerline radius of 4,000 ft. For this flow:

(1) Calculate the mean transverse bed slope.

(2) Calculate and plot the convex bed slope.

(3) Calculate the lateral distribution of unit discharge, V(r)d(r), and compare $\int vd \cdot dr$ (integrated numerically) with the discharge given. Assume f is constant across the channel.

(4) Calculate the meander wave length.

Note: Use D_g in all calculations.

<PROBLEM K-2M>

For the data given in Problem K-1T, calculate the following for a long river bend with r_c = 1,500 ft.

(1) The depths near the concave and the convex banks.

(2) The velocities near the concave and the convex banks. Please base your calculations on the given value of D'_{50}, and assume the following values: θ = 0.06 and p = 0.45

<PROBLEM K-3M>

A bend in a river has width B and centerline radius $r_c \geq B/2$. The vertical distribution of streamwise velocity is given by

$$u(y) = \frac{2u_s}{d}\left[y - \frac{y^2}{2d}\right] \qquad (1)$$

which corresponds to constant eddy viscosity. The bed material has density ρ_s and uniform diameter D. Derive an equation for the lateral (radial) bed slope, S_t, in terms of u_s (surface velocity) and the other pertinent variables. Assume that the radial slope is sufficiently small that the section may be treated as rectangular in certain respects, and assume that the bend radius is constant (i.e., the centerline of the channel is circular).

<PROBLEM K-4M>

A curved river with semi-circular cross section of radius r_o has a constant channel centerline radius R. Calculate the average radial-plane shear stress, τ_r.

<PROBLEM K-5M>

On 2 May 1978 the following data were obtained for the Sacramento River near Chico, California:

Water discharge	Q	=	7,780 cfs;
Centerline radius of curvature	r_c	=	1,500 ft;
Channel width	B	=	535 ft;
Flow depth (average)	d	=	9.0 ft;
Median sediment size	D_{50}	=	0.14 mm; and,
Bed friction factor	f_b	=	0.08

Calculate the average transverse bed slope of the stream, and the maximum and minimum depths across the channel.

<PROBLEM N-6M>

The net radial momentum flux, M_R, through the small control volume in the polar-coordinate system of bend flow consists of the streamwise difference in radial momentum passing through the upstream and downstream faces, and the radial difference in radial momentum passing through the inside and outside faces, and is given by

$$M_R = \frac{\partial}{\partial \theta}\left\{\int_{z_b}^{z_s}(\rho v)u \, dRdz\right\}d\theta + \frac{\partial}{\partial R}\left\{\int_{z_b}^{z_s}\rho v^2 R \, d\theta dz\right\}dR \tag{1}$$

Show that

$$M_R = \frac{\partial}{\partial \theta}\left\{\rho U(z_s - z_b)\frac{V}{(2n+1)}dR\right\}d\theta + \frac{\partial}{\partial R}\left\{\rho R d\theta (z_s - z_b)\frac{V^2}{3}\right\}dR \tag{2}$$

where

$v =$ linear secondary flow velocity in the R direction $= 2V[(z-z_b)/h-1/2]$;

$u =$ local streamwise velocity based on a power-law relationship $=$ $u_s[(z- z_b)/h]^{(1/n)} = U(n+1)/n)[(z-z_b)/h]^{(1/n)}$;

z_s = water-surface elevation; and,

z_b = bed elevation